UNIT - I

8th oct

MATHEMATICAL LOGIC.

Propositional Logic (or) Statements :- A Declarative Sentence is said to be Statement if it is either true or false.

In a statement true value is denoted by 'T' and false value is denoted by 'F'.

Statements are denoted by small letters i.e., p, q, r, s....

Ex:- 1. India is a country [T]
∴ it is a statement

2. Rose is red but not flower [F]
∴ it is a statement.

3. Hyderabd is a pink city [F]
∴ it is a statement.

4. 4+3=7 [T] it is a statement.

(1)

Connectivities :- Two statements are connected to some terms are called connectivities. ie; they are and, or, then, if, iff $\Leftrightarrow$, not.

Compound Statements :- Two statements are connected to connectivities, we get New statement. the new statement is said to be compound statement.

Ex:- 4+5=9 and 3+6=9
4+5=9 or 3×4=7
x+4=5 if x=1
4+3=7 iff 3+4=7.

Types of Compound Statements :-

1. Conjuction
2. disjuction
3. conditional
4. Bi-conditional

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5. Negation. (or) Negation Statement.

1. Conjuction Statement :— Two statements are connected to AND (∧). Then compound statement is called conjuction. It is denoted by "$p \wedge q$." where p, q are two statements.

Property :— 'p' is a True [T], 'q' is a True [T] then compound statement is also True [T]. otherwise it is false.

Truth table :—

p	q	$p \wedge q$.
T	T	T
T	F	F
F	T	F
F	F	F.

(2)

Ex :— 1. P: Sree vani clg is a girls clg

q: Sree vani clg is a women's clg.

$p \wedge q$: Sree vani clg is a girls and women's clg.

∴ It is a conjuction.

2. Sree vani clg is a girls clg and boys PG clg.

∴ It is not a conjuction.

2. Dijuction Statement :— Two statements are connected to OR (∨). Then compound statement is called dijuction. It is denoted by "$p \vee q$." where p, q are two statements.

Property :— 'p' is a false [F] and 'q' is a false [F] then compound statement is false [F] otherwise it is true [T]

Truth table :—

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

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Ex:- 1. P : Sree vani clg is a girls clg.

Q : Sree vani clg is a women's clg.

P∨Q :

∴ It is a disjuction

2. P : Sree vani is a girls clg. (3)

Q : Sree vani is a boys clg.

∴ It is also disjuction

3. P : Sree vani clg. is a boys clg.

Q : Sree vani clg is a co-ed clg.

∴ It is also disjuction

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3. Conditional Statement :- Two statements are connected to if, then. Then compound statement is called condition statement. and it is denoted by $P \Rightarrow Q$.

Property :- If P is true [T] Q is false [F] then compound statement is false [F] otherwise it is true [T]

Truth table :-

P	Q	P⇒Q
T	T	T
T	F	F
F	T	T
F	F	T

4. Bi-Conditional Statement :- Two statements are connected to if and only iff., then compound statement is called Bi-conditional statement. It is denoted by $P \Leftrightarrow Q$.

Property :- If P is a true [T] and Q is ~~False~~ True [F] then compound statement is also True [T].

If P is false [F] and Q is False [F] then compound statement is also True [T] otherwise it is

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false [F].

Truth table :-

P	q	$P \Leftrightarrow q$
T	T	T
T	F	F
F	T	F.
F	F	T

Negation :- (or) Not Statement :- The opposite of statemen is called Negation (or)

The statement before we apply NOT or No. then compound statement is called Negation. It is denote by '~' (or) ⌐.

If P is a statement then Negation of 'P' is Negation 'P'.

Note :- 1. $\sim(\sim P) = P$

2. $\sim(\sim)(\sim P) = \sim P$

Problems :-

I. write the truth tables of the following statements.

1) $\sim(P \Rightarrow q) \wedge (q \Rightarrow P)$

Sol :-

P	q	$P \Rightarrow q$	$q \Rightarrow P$	$\sim(P \Rightarrow q)$	$\sim(P \Rightarrow q) \wedge (q \Rightarrow P)$
T	T	T	T	F	F
T	F	F	T	T	T
F	T	T	F	F	F
F	F	T	T	F	F.

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2) write the truth table of compound statement

$(P \vee \sim q) \rightarrow (P \wedge q)$

Sol:-

P	q	~q	P∧q	P∨~q	(P∨~q)→(P∧q)
T	T	F	T	T	T
T	F	T	F	T	F
F	T	F	F	F	T
F	F	T	F	T	F

3) $P \wedge \sim P$.

(5)

Sol:-

P	q	~P	P∧~P
T	T	F	F
T	F	F	F
F	T	T	F
F	F	T	F

4) $P \vee \sim P$

Sol:-

P	q	~P	P∨~P
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	T

5) $(P \vee \sim q) \rightarrow q$

Sol:-

P	q	~q	P∨~q	(P∨~q)→q
T	T	F	T	T
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

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6. $(p \vee q) \rightarrow (p \wedge q)$

p	q	$p \vee q$	$p \wedge q$	$(p \vee q) \rightarrow (p \wedge q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

(6)

7. $(p \rightarrow q) \Leftrightarrow (\sim p \rightarrow \sim q)$

p	q	$\sim p$	$\sim q$	$p \rightarrow q$	$\sim p \rightarrow \sim q$	$(p \rightarrow q) \Leftrightarrow (\sim p \rightarrow \sim q)$
T	T	F	F	T	T	T
T	F	F	T	F	T	F
F	T	T	F	T	F	F
F	F	T	T	T	T	T

8. $(p \rightarrow q) \rightarrow (q \rightarrow p)$

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \rightarrow (q \rightarrow p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

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Note :-

1. If P, Q are two statements then true values are four i.e; $2^2 = 4$ (7)
2. If P, Q, R are 3 statements then truth values are $2^3 = 8$.

* Converse, Inverse & contra position of a conditional statement.

If P, q are two statement.

1. conditional statement :- $p \Rightarrow q$
2. converse statement :- If $p \Rightarrow q$ is a conditional then $q \Rightarrow p$ is called converse statement.
3. Inverse :- If $p \Rightarrow q$ is a conditional statement then $\sim p \Rightarrow \sim q$ is called Inverse statement,
4. Contra position :- If $p \Rightarrow q$ is a Conditional statement then $\sim q \Rightarrow \sim p$ is called Contraposition.

→ Truth table :- Truth table of Converse, Inverse, & Contra position of condition statement.

P	q	~p	~q	p⇒q	q⇒p	~p⇒~q	~q⇒~p
T	T	F	F	T	T	T	T
T	f	F	T	f	T	T	F
F	T	T	F	T	f	f	T
f	F	T	T	T	T	T	T

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Note :- 1. Conditional statement Equivalent to contraposition statement

2. Converse statement is Equivalent to Inverse statement.

Ex:- In a ΔABC, $AB = Ac$ then $\angle B = \angle C$. (8)

Given statement : In ΔABC, $AB = Ac$ then $\angle B = \angle C$.

$P: AB = Ac$, $q: \angle B = \angle C$.

Conditional : $P \Rightarrow q$ In ΔABC, $AB = Ac$ then $\angle B = \angle C$.

Converse : $q \Rightarrow p$ In ΔABC, $\angle B = \angle C$ then $AB = Ac$.

Inverse : $\sim p \Rightarrow \sim q$, In ΔABC, $AB \neq Ac$ then $\angle B \neq \angle C$

Contra position : $\sim q \Rightarrow \sim p$ In ΔABC, $\angle B \neq \angle C$ then $AB \neq Ac$.

Ex 2:- P: Hyd is a city, q: Hyd has a large pollution. then we write conditional, converse, Inverse and contraposition statement.

Sol:- Conditional :- $P \Rightarrow q$ Hyd is a city then Hyd has a large pollution.

Converse :- $q \Rightarrow p$, Hyd has a large pollution then Hyd is a city.

Inverse :- $\sim p \Rightarrow \sim q$. Hyd is not a city then Hyd has no large pollution.

Contraposition :- $\sim q \Rightarrow \sim p$. Hyd has no large pollution then Hyd is not a city.

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Equivalent Statement or proportional Statement :—

Tautology :— A compound Statement is called Tautology. it has only capital T values or all values are truth (T).

Example :—

P	~P	P∨~P
T	F	T
F	T	T.

Contradiction :— A compound Statement is called contradiction. if it has all the values false (F).

Example :—

P	~P	P∧~P
T	F	F
F	T	F.

Equivalent Statement (or) Equal Statements :—

Two compound Statement truth values are Equal then they are called Equivalent & Equal Statement.

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Problems:- 23rd Oct

1) To verify the following statements $p \wedge q \Leftrightarrow q \wedge p$. (8)

1) $p \wedge q \cong q \wedge p$.

Sol:- $p \wedge q$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F.

$q \wedge p$ (10)

p	q	$q \wedge p$
T	T	T
T	F	F
F	T	F
F	F	F.

$\therefore p \wedge q \cong q \wedge p$. /—

2) $\sim (p \wedge q) \equiv \sim p \vee \sim q$.

Sol:- $\sim (p \wedge q)$

p	q	$p \wedge q$	$\sim (p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

$\sim p \vee \sim q$

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

$\therefore \sim (p \wedge q) \cong \sim p \vee \sim q$. /—

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3) $\sim(p \vee q) \cong \sim p \wedge \sim q$.

Sol:- $\sim(p \vee q)$

p	q	$p \vee q$	$\sim(p \vee q)$
T	T	T	F
T	F	F	F
F	T	F	F
F	F	F	T.

$\sim p \wedge \sim q$

p	q	$\sim p$	$\sim q$	$\sim p \wedge \sim q$.
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

$\therefore \sim(p \vee q) \cong \sim p \wedge \sim q$ /—

2) By using truth tables to verify the following statements are Equivalence.

1. $p \wedge T \cong p$ (ii) $p \vee f \cong p$ (iii) $p \wedge f \cong f$ (iv) $p \vee T \cong T$

(v) $p \vee p \cong p$ (vi) $p \wedge p \cong p$.

Sol:- 1. $p \wedge T \cong p$

p	T	$p \wedge T$
T	T	T
T	T	T
F	T	F
F	T	F

$\therefore p \wedge T \cong p$.

iii) $p \vee F \cong p$

p	F	$p \vee F$
T	F	T
T	F	T
F	F	F
F	F	F

$\therefore p \vee F \cong p$

(iii) $p \wedge F \cong F$

p	F	$p \wedge F$
T	F	F
T	F	F
F	F	F
F	F.	F.

$\therefore p \wedge F \cong F$.

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(iv) $P \vee T \cong T$

P	T	P ∨ T
T	T	T
T	T	T
F	T	T
F	T	T

$\therefore P \vee T \cong T$

(v) $P \vee P \cong P$

P	P	P ∨ P
T	T	T
T	T	T
F	F	F
F	F	F

$\therefore P \vee P \cong P$

(vi) $P \wedge P \cong P$

P	P	P ∧ P
T	T	T
T	T	T
F	F	F
F	F	F

$\therefore P \wedge P \cong P$.

3) S.T P bi-implies q is Equivalent to (P and q). or, Negation P and Negation q.

S.T $P \leftrightarrow q \cong (P \wedge q) \vee (\sim P \wedge \sim q)$ (or)

$P \leftrightarrow q \cong (P \wedge q) \vee (\sim P \wedge \sim q)$

Sol:- L.H.S. $P \leftrightarrow q$.

P	q	P ↔ q
T	T	T
T	F	F
F	T	F
F	F	T

R.H.S. $(P \wedge q) \vee (\sim P \wedge \sim q)$

P	q	~P	~q	P ∧ q	(~P ∧ ~q)	(P ∧ q) ∨ (~P ∧ ~q)
T	T	F	F	T	F	T
T	F	F	T	F	F	F
F	T	T	F	F	F	F
F	F	T	T	F	T	T

$\therefore P \leftrightarrow q \cong (P \wedge q) \vee (\sim P \wedge \sim q)$ /-

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4) S.T. $\sim(p \to q) \cong p \leftrightarrow \sim q$.

Sol:-

p	q	$p \to q$	$\sim(p \to q)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F.

(13)

$p \leftrightarrow \sim q$

p	q	$\sim q$	$p \leftrightarrow \sim q$.
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

$\therefore \sim(p \to q) \not\equiv p \leftrightarrow \sim q$. //

5) S.T. $(p \to q) \wedge (p \to r) \cong p \to (q \wedge r)$

Sol:- L.H.S.

$(p \to q) \wedge (p \to r)$

p	q	r	$p \to q$	$p \to r$	$(p \to q) \wedge (p \to r)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	F
T	F	F	F	F	F
F	T	T	T	T	T
F	T	F	T	T	T.
F	F	T	T	T	T
F	F	F	T	T	T

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RHS:- $p \rightarrow (q \wedge r)$

p	q	r	$q \wedge r$	$p \rightarrow (q \wedge r)$
T	T	T	T	T
T	T	F	F	F
T	F	T	F	F
T	F	F	F	F
F	T	T	T	T
F	T	F	F	T
F	F	T	F	T
F	F	F	F	T

(14)

$\therefore (p \rightarrow q) \wedge (p \rightarrow r) \cong p \rightarrow (q \wedge r)$ /—

24th Oct

Formulas :-

1. Commutative Law :- $p \wedge q \cong q \wedge p$, $p \vee q \cong q \vee p$.
2. Idempotal Law :- $p \wedge p \cong p$, $p \vee p \cong p$.
3. Associative Law :- $(p \wedge q) \wedge r = p \wedge (q \wedge r)$,
   $(p \vee q) \vee r = p \vee (q \vee r)$
4. Distributive Law :- $p \wedge (q \vee r) \cong (p \wedge q) \vee (p \wedge r)$,
   $p \vee (q \wedge r) \cong (p \vee q) \wedge (p \vee r)$
5. Demorgan's Law :- $\sim(p \wedge q) \cong \sim p \vee \sim q$,
   $\sim(p \vee q) \cong \sim p \wedge \sim q$.
6. double Negation Law :- $\sim(\sim p) \cong p$
7. $p \wedge \sim p \cong F$,
   $p \wedge F \cong F$, $p \vee T \cong T$, $p \vee F \cong p$
8. $p \wedge (p \vee q) \cong p$.
   $p \vee (p \wedge q) \cong p$.

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1) S.T $\sim[p \vee (\sim p \wedge q)]$ and $\sim p \wedge \sim q$ are Equivalent with Truth tables

Sol:- $\sim[p \vee (\sim p \wedge q)]$

$\sim p \wedge \sim(\sim p \wedge q)$

$\sim p \wedge [\sim(\sim p) \vee \sim q)$ (15)

$\sim p \wedge (p \vee \sim q)$

$(\sim p \wedge p) \vee (\sim p \wedge \sim q)$ $[\because \sim p \wedge p = F]$

$F \vee (\sim p \wedge \sim q)$

$\therefore \sim p \wedge \sim q$ /-

2. S.T $\sim[p \vee (\sim p \vee q)]$ and F are Equivalence.

Sol:- $\sim[p \vee (\sim p \vee q)]$

$\sim p \wedge \sim(\sim p \vee q)$

$\sim p \wedge ((\sim) \sim p \wedge \sim q)$

$\sim p \wedge (p \wedge \sim q)$

$(\sim p \wedge p) \wedge (\sim p \wedge \sim q)$

$F \wedge (\sim p \wedge \sim q)$

$\therefore F$ /-

Functionaly complete set of NAND, NOR :-

NAND :- The Negation of conjuction is called NAND Statement.

Truth table :-

P	q	$p \wedge q$	$\sim(p \wedge q)$ (or) NAND (or) $p \uparrow q$.
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

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NOR :- The negation of disjuction is called NOR statement.

Truth table :-

P	q	pvq	~(pvq) = NOR = P↓q
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

(16)

Boolean Algebra :- Complement of lattice is called Boolean Algebra. and it is denoted by $(B, *, \oplus, 0, 1)$

Binary Operations :- If $(*, \oplus)$ are called binary operations

Unitary operations :- If $(0, 1)$ are called unitary operators. where '0' is the least value and 1 is the greatest value.

Complement of a unitary operators :- The compliment of 1 is $\bar{1} = 0$

Compliment of $\bar{0}$ is $\bar{0} = 1$

Boolean sum :-

$1+1 = 1$
$1+0 = 1$
$0+1 = 1$
$0+0 = 0$

Boolean Product :-

$1 \times 1 = 1$
$1 \times 0 = 0$
$0 \times 1 = 0$
$0 \times 0 = 0$

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Properties of Boolean Algebra:—

1. Commutative law:— $a*b = b*a$, $a \oplus b = b \oplus a$.
2. Associative law:— $(a*b)*c = a*(b*c)$

   $(a \oplus b) \oplus c = a \oplus (b \oplus c)$
3. Idepontal Law:— $a*a = a$, $a \oplus a = a$ (17)
4. Distributive law:— $a*(b \oplus c) = (a*b) \oplus (a*c)$ is Called left Distributive law.

   $(b \oplus c)*a = (b*a) \oplus (c*a)$ is Called right distributive law.

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The relation is defined by B such that $(B, \leq)$ then

1. $(a*b)$ = greatest lower bound of (A, B)
2. $(A \oplus b)$ = lower upper bound of (A, B)
3. $(a \leq b) \Leftrightarrow (a*b) \Leftrightarrow (a \oplus b) = B$
4. $A \leq B \Leftrightarrow (a*b') = 0 \Leftrightarrow \bar{b} \leq \bar{a} \Leftrightarrow \bar{a} \oplus \bar{b} = 1$
5. Let $(B, *, \oplus, 0, 1)$ is a Boolean Algebra. and $A, \bar{A} \in B$ then
   1. $a*\bar{a} = 0$
   2. $\overline{(a*b)} = \bar{a} \oplus \bar{b}$
   3. $a \oplus \bar{a} = 1$
   4. $\overline{(a \oplus b)} = \bar{a}*\bar{b}$
   5. $0 \leq a \leq 1$
   6. $a*0 = 0$
   7. $a \oplus 1 = 1$
   8. $a*1 = a$
   9. $a*0 = 0$.

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1. P.T. $[a*(\overline{b \oplus c})] * [\overline{b \oplus (a*c)}] = a*b*\bar{c}$

(8)

$[a*(b' \oplus c)]' * [\overline{b \oplus (a*c')}]' = a*b*c'$

(8) (18)

$[\overline{x*y' \oplus z}] * [\overline{y \oplus (x*z)}] = x*y*\bar{z}$.

Sol:- L.H.S.

$[a*(b' \oplus c)]' * [b' \oplus (a*c')]'$

$a' \oplus (b' \oplus c')' * [(b')' * (a*c')']$

$a' \oplus [(b')'*c'] * b * [a' \oplus (c')']$

$a' \oplus [b*c'] * b * (a' \oplus c)$

$[a' \oplus b] * [a' \oplus c'] * [(b*a') \oplus (b*c)]$

∵ We know that. $(a*b) \oplus (b*c) \oplus (c*a) = (a \oplus b) * (b \oplus c) * (c \oplus a)$

and. $(a \oplus b) * (b \oplus c) * (c \oplus a) = (a*b) \oplus (b*c) \oplus (c*a)$

$\therefore [a'*b] * [a'*c'] \oplus [(b \oplus a') * (b \oplus c)]$

$\therefore 0*b*c' \oplus b*a*c' = 0 \oplus b*a*c'$

$\therefore a*b*c'$ /—

2. S.T $a=b \Rightarrow (a*b') \oplus (a'*b) = 0$ (8)

S.T. $a=b \Rightarrow (a*b') \oplus (a'*b) = 0$.

Sol:- Let $a=b$.

Now we have to s.t $(a*b') \oplus (a'*b)$ = [∵ $a=b$]

$(a*a') \oplus (a'*a)$

$0 \oplus 0$

0 /—

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3. Simplify $(\bar{a} * \bar{b} * c) \oplus (a * \bar{b} * c) \oplus (a * \bar{b} * \bar{c})$.

Sol:- $(\bar{a} * \bar{b} * c) \oplus (a * \bar{b} * c) \oplus (a * \bar{b} * \bar{c})$

~~$(\bar{a} \oplus \bar{b} \oplus c) * (a \oplus \bar{b} \oplus c) * (a \oplus \bar{b} \oplus \bar{c})$~~

L.H.S $[(\bar{a} * c) \oplus (a * c) \oplus (a * \bar{c})] * \bar{b}$.

$[(\bar{a} * c) \oplus [a * (c \oplus \bar{c})] * \bar{b}$ $[\because c \oplus \bar{c} = 1]$

$[(\bar{a} * c) \oplus [a * 1] * \bar{b}$.

$(\bar{a} * c) \oplus a * \bar{b}$.

$(\bar{a} \oplus a) * c \oplus a * \bar{b}$ $[\because \bar{a} \oplus a = 1]$

$1 * (c \oplus a) * \bar{b}$

$c \oplus a * \bar{b}$. RHS.

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4. P.T following Boolean identifications.

1. $a \oplus (a' * b) = a \oplus b$

Sol:- L.H.S $a \oplus (a' * b)$

$(a \oplus a') * (a \oplus b)$

$1 * (a \oplus b)$

$a \oplus b$

2. $(a * b * c) \oplus (a * b) = a * b$

Sol:- L.H.S. $(a * b * c) \oplus (a * b)$

$(a * b) * [c \oplus 1]$

$(a * b) * 1$

$(a * b)$

3. If $a = 0$ then $ab' + a'b = a$

Sol:- L.H.S. $ab' + a'b$

$0b' + 0'b$

$0 + 0$

$= 0$

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4. P.T. $(a*b) \oplus (a'*b) = b$

Sol:- L.H.S. $(a*b) \oplus (a'*b)$

$[a \oplus a'] * b$

$1 * b$

$= b$. /—

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5) Simplify $(a*b')' \oplus (a \oplus b)'$

Sol:- L.H.S

$(a*b')' \oplus (a \oplus b)'$

$(a'*b') \oplus (a' \oplus b')$

[$\because a * \bar{a} = 0$ then $b' * b' = 1$]

$a' \oplus [b' * b']$

$a' \oplus 1$

a'. /—

Boolean functions :-

Boolean variable :- Let $B = \{0, 1\}$ and $B^n = \{x, x^2 \ldots x^n\}$ then x values are called Boolean variables.

Boolean function :- A mapping from $B \to B^n$ or $B^n \to B$ is called Boolean function and it is denoted by $F: B \to B^n$

1. find the value of $1.0 + \overline{(0+1)}$

Sol:-

$1.0 + \overline{(0+1)}$

$0 + \bar{1}$

$0 + 0$

0 /—

2. find the values of 1. $1.\bar{0}$

2. $1+\bar{1}$

3. $\bar{0}.0$

4. $[\overline{1+0}]$

Sol:- (1) ~~$1+\bar{0}$~~ Given that

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1. $1 \cdot \bar{0}$ | $1+\bar{1}$ | $\bar{0} \cdot 0$ | $[1+\bar{0}]$

$1 \cdot 1$ | $1+0$ | $1 \cdot 0$ | $1+0$

$= 1$ | 1 | 0 | 1 /—

26th oct

3). $1.0 + (\bar{0}+1)$ convert to logically Equivalent (or) Explain $1.0+(\bar{0}+1)$ in logically (statement) Equivalence.

Sol:- $1.0 + (\bar{0}+1)$

$(T \wedge F) \vee \sim(F \vee T)$ (21)

$F \vee [\sim F \wedge \sim T]$

$[F \vee \sim F] \wedge (F \vee \sim T)$

$(F \vee T) \wedge (F \vee F)$

$T \wedge F$

$\therefore F$ /—

4. Find the values of Boolean function by $f(x,y) = xy$ by using table.

Sol:-

x	y	xy
1	1	1
1	0	0
0	1	0
0	0	0.

/—

5. find the values of the Boolean function $f(x,y,z) = xy + \bar{z}$

Sol:-

x	y	z	xy	$\bar{z}$	$xy+\bar{z}$.
1	1	1	1	0	1
1	1	0	1	1	1
1	0	1	0	0	0
1	0	0	0	1	1
0	1	1	0	0	0
0	1	0	0	1	1
0	0	1	0	0	0
0	0	0	0	1	1

/—

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6. use a table to find the values of following Boolean function.

Sol:- 1. $f(x,y,z) = \bar{x} \cdot y$ 2. $f(x,y,z) = x + yz$.

3. $f(x,y,z) = x\bar{y} + (\overline{xyz})$ 4. $f(x,y,z) = \bar{y}(yz + \bar{y}z)$

1Sol:- $f(x,y,z) = \bar{x} \cdot y$

x	y	z	$\bar{x}$	$\bar{x}y$
1	1	1	0	0
1	1	0	0	0
1	0	1	0	0
1	0	0	0	0
0	1	1	1	1
0	1	0	1	1
0	0	1	1	0
0	0	0	1	0

2Sol:- $f(x,y,z) = x + yz$.

x	y	z	yz	$x+yz$
1	1	1	1	1
1	1	0	0	1
1	0	1	0	1
1	0	0	0	1
0	1	1	1	1
0	1	0	0	0
0	0	1	0	0
0	0	0	0	0

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Sol:- $q(x,y,z) = x\bar{y} + (x\bar{y}z)$

x	y	z	$\bar{y}$	$x\bar{y}$	xy	xyz	$x\bar{y}z$	$x\bar{y} + (x\bar{y}z)$
1	1	1	0	0	1	1	0	0
1	1	0	0	0	1	0	1	1
1	0	1	1	1	0	0	1	1
1	0	0	1	1	0	0	1	1
0	1	1	0	0	0	0	1	1
0	1	0	0	0	0	0	1	1
0	0	1	1	0	0	0	1	1
0	0	0	1	0	0	0	1	1

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Sol:- $q(x,y,z) = \bar{y}(yz + \bar{y}z)$

x	y	z	$\bar{y}$	yz	$\bar{y}z$	$(yz + \bar{y}z)$	$\bar{y}(yz + \bar{y}z)$
1	1	1	0	1	0	1	0
1	1	0	0	0	0	0	0
1	0	1	1	0	1	1	1
1	0	0	1	0	0	0	0
0	1	1	0	1	0	1	0
0	1	0	0	0	0	0	0
0	0	1	1	0	1	1	1
0	0	0	1	0	0	0	0

$P \rightarrow [q \rightarrow (p \wedge q)]$

p	q	$p \wedge q$	$q \Rightarrow p \wedge q$	$P \rightarrow [q \rightarrow (p \wedge q)]$
T	T	T	T	T
T	F	F	T	T
F	T	F	F	T
F	F	F	T	T

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Double complement Law:—

1. $\bar{\bar{x}} = x$
2. $x+x = x$, $x \cdot x = x$,
3. $x+0 = x$, $x+1 = 1$, Identity law
4. $x+y = y+x$, $xy = yx$. Commutative law
5. $x+y+z = (x+y)+z$, $x(yz) = (xy)z$. Associative law
6. Distributive Law:—
   $x(y+z) = xy + xz$.
   * $(x+y)(x+z) = x + yz$.
7. Demorgance Law:— $\overline{xy} = \bar{x} + \bar{y}$, $\overline{(x+y)} = \bar{x} \cdot \bar{y}$.
8. unit Law:— $x + \bar{x} = 1$, $x \cdot \bar{x} = 0$, $x+1 = 1$, $x \cdot 0 = 0$,
   $x + xy = x$

1. P.T $x \cdot (x+y) = x$ by using boolean algebra formulas.

Sol:— L.H.S $x(x+y) = (x+0)(x+y)$

$= (x+y)(x+0)$ [$\because (x+y)(x+z) = x+yz$]

$= x + y(0)$

$= x+0$

$= x$ RHS/—

Dual Boolean Expression:— The Dual of Boolean Expression is obtained by Interchanging Boolean sum and Boolean product. i.e., Interchanging Zero's & ones.

→ Boolean Sum = '+'.
→ Dual Boolean Expression = '·'.
→ Boolean Product = '·'
→ Dual Boolean Expression = '+'.

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→ Boolean value = 0.
→ Dual Boolean Expansion = 1.
→ Boolean value = 1
→ Dual. Boolean Expansion = 0

1) find the duals Expansion of $x.(y+0)$ and $\bar{x}.1+(\bar{y}+z)$

Sol:- $x(y+0)$

Dual of $x(y+0) = x+(y.1)$;
$= x+y.$

$\bar{x}.1+(\bar{y}+z)$
$(\bar{x}+0).(\bar{y}.z)$
$\bar{x}.\bar{y}.z$
$\bar{x}\bar{y}z$ /—

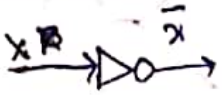
Logic Gates :- The basic Elements of circuits are called logic gates.

Types of gates :-

1. Inverter. 2. OR gate 3. AND gate.

1. Inverter :- If x is a Element and $\bar{x}$ is a result of x.

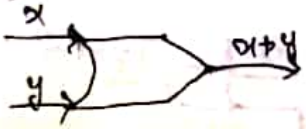
∴ 'x' is a input Element and $\bar{x}$ is a output Element & leaving Element.

Symbol of Inverter :- 

2. OR gate :- If x, y are two Elements then $x+y$. is also Element.

x, y are input elements and $x+y$ is output Element.

Symbol of OR gate :-



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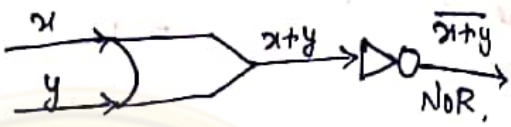
. AND gate :- If x, y are two statements then xy is also Element.

x, y are input Elements and xy is output Element.

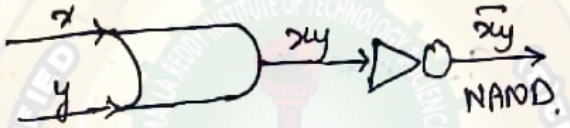
Symbol of AND gate :-

Note :-

1. NOR Gate :-

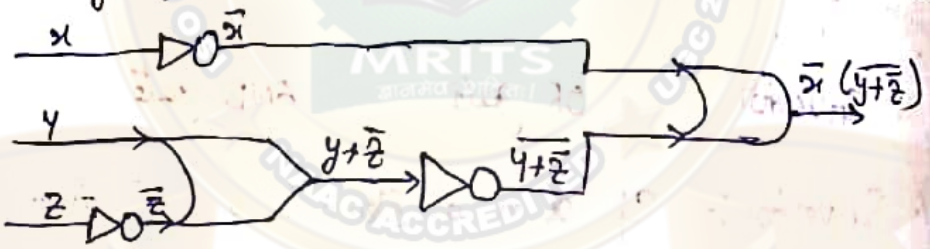


2. NAND Gate :-

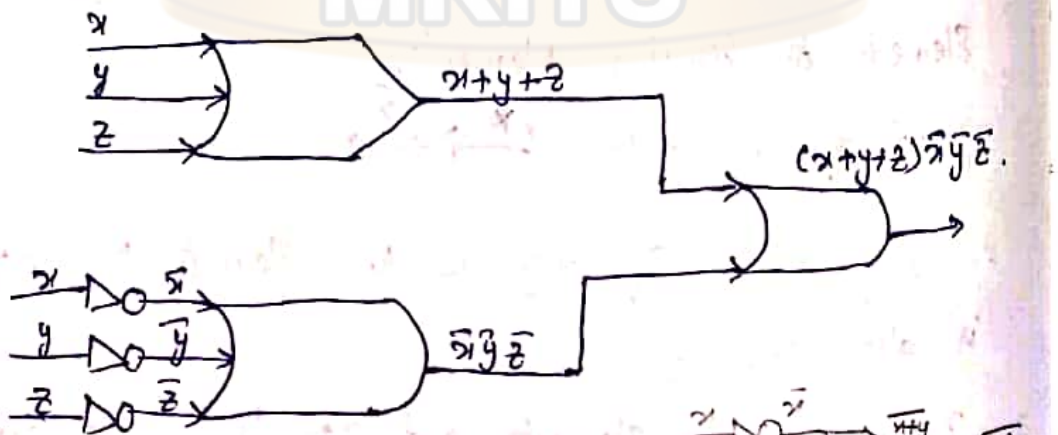


(1) Problem :-

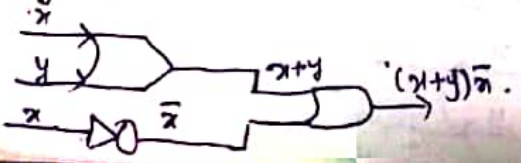
2) $\bar{x}(\overline{y+\bar{z}})$

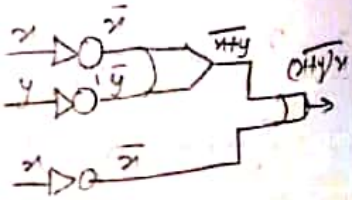


3) $(x+y+z)\,\bar{x}\,\bar{y}\,\bar{z}$.



1. $(x+y)\,\bar{x}$.





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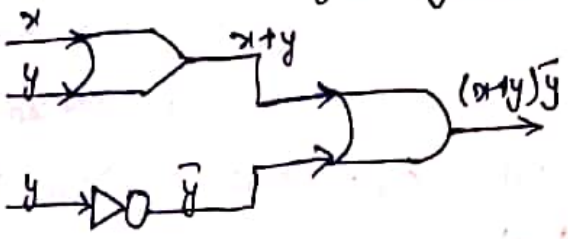
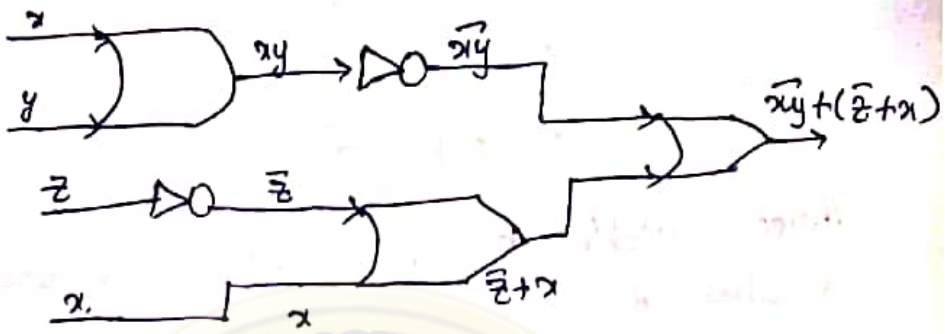
4) find the output of the given figures :-

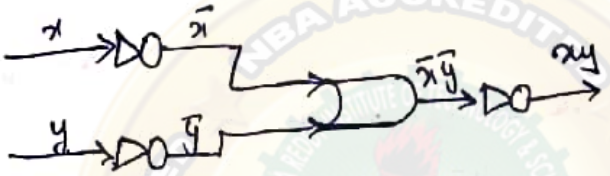
i)



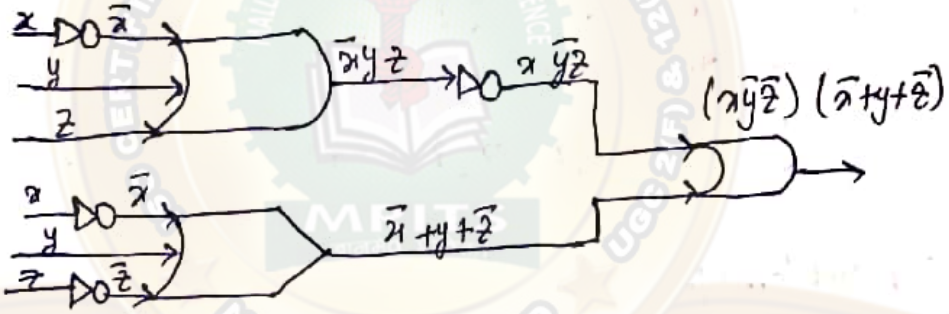
ii)



iii)



iv)



Minimization of circuit :-

find the sum of the product is called minimization of the boolean function.

Ex :- $\Sigma xy = xy + yz + zx$

K- Map :-

The boolean Expansion representing in a graphical method is called k-map.

→ Types of k-maps

1. Two variables k-map.
2. Three variables k-maps.

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Two variables k-map:- Let x, y are two variables of a k-map. for a boolean function in those two variables consider of two cells when all is placed in the cell representing a minimum. If it is minimum is present in the expressions.

	y	$\bar{y}$
x	xy	$x\bar{y}$
$\bar{x}$	$\bar{x}y$	$\bar{x}\bar{y}$

Three variables k-map:- Let x, y, z are three variables of a k-map. In three variables is rectangle divided into 8 cells. The cells represents the 8 possible minimum three variables in this k-map.

	yz	$y\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}z$
x	xyz	$xy\bar{z}$	$x\bar{y}\bar{z}$	$x\bar{y}z$
y	$\bar{x}yz$	$\bar{x}y\bar{z}$	$\bar{x}\bar{y}\bar{z}$	$\bar{x}\bar{y}z$

Problems:-

1) use k-mapping to find minimization. (or) find the sum of product of the following functions.

$f(a,b,c,d) = f(x,y,z,t) = \Sigma(0,2,6,7,8,9,13,15)$

Sol:-

0	2	6	7	8	9	13	15
	2|2 P-0	2|6	2|7	2|8	2|9	2|13	2|15
		2|3-0	2|3-1	2|4-0	2|4-1	2|6-1	2|7-1
	0010	1-1	1-1	2|2-0	2|2-0	2|3-0	2|3-1
		0110	0111	1-0	1-0	2|1-1	2|1-1
				1000	1001	1101	1,1
							1111

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UNIT - II

31st Oct

SETS.

Sets:- The collection of finite or infinite items & objectives are called set. and it is denoted by A, B, C...

Ex:- A = {1, 2, 3, 4}

B = {1, 2, 3, 4 ....}

Elements:- In a set the items are called elements and it is ~~data~~ denoted by a, b, c .....

finite set:- A set is called finite set. If it is countable set.

Ex:- A = {1, 2, 3, 4}

B = {a, e, i, o, u}

Infinite set:- A set is called infinite set If it is not a finite set.

Ex:- A = {1, 2, 3 ....}

Symbol of set:- The set is denoted by circle [O] or triangle [Δ].

Set forms:- They are two types.

1. Roster form. 2. listing (or) building form.

Roster form:- The elements are separated by commas [,] then the form is called Roster form.

Ex:- A = {a, b, c}

listing form:- The elements are arranged in the form of {x / x ∈ A} is called listing or building form.

Ex:- A = {y = 2x / x ∈ z}

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Example of finite set :- A = { a, b, c, d } B = { Comp. Science Stud
C = { Pure math Students } D = { set of computer, math Students }

Examples of Infinite set :- N, W, I, Q, Q', R, C

Null-Set (OR) Empty set :- A set is said to be Null set. If it is not containing Elements and it is denoted by ∅ or { } Empty set.

Therefore Null-Set = ∅ or { }

Note :- {0} is not a null set.

Single set :- A set is said to be single set if it is containing only one Element.

Ex :- {1}, {a}, {0}

Subset :- If A, B are two non-Empty sets such that Every Element of 'A' belongs to set 'B'. Then 'A' is called subset of 'B' and it is denoted by $A \subseteq B$

Ex :- A = {1, 2, 3, 4}, B = {2, 3} C = {2}

$B \subseteq A$. $C \subseteq A$ & $C \subseteq B$.

A = { Sree vani clg } B = { comp. SC Students }

C = { Pure math Students }

$\therefore B \subseteq A$ & $C \subseteq A$.

Equal sets :- Let A, B are two sets such that Every Element of A is Equal to Every Element of B. then they are called Equal sets.

Ex :- 1. A = {1, 2, 3}, B = {2, 3, 1} $\therefore A = B$.

2. A = { comp. science students }, B = { Pure math Students }

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$\therefore A \neq B$.

3. $A = \{$ Comp. Science Dm syllabus $\}$

$B = \{$ Pure. math Dm Syllabus $\}$ $\therefore A = B$

Note :- Two sets A, B such that $A \subseteq B$ and $B \subseteq A$. Then. $A = B$

CORTIGIAN numbers :- The no. of Elements in a set. 'A' is called cortigian numbers and it is denoted by 'n(A)'.

Ex :- 1. $A = \{1, 2, 3\}$

$\therefore n(A) = 3$.

2. The C.N. of set of natural number is.

$\therefore n(A) =$ Infinite (∞)

Note :- $n(A) = n$, then. no. of subsets $= 2^n$.

Ex :- $A = \{1, 2, 3\}$

$\therefore n(A) = 3$.

$\therefore$ no. of subsets $= 2^3 = 8$.

$\therefore$ Subsets are. $\{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{3,1\}, \{1,2,3\}, \emptyset$.

Power set :- The set of all subsets is called. Power set. and it is denoted by P(A).

3rd Nov

Ex :- $\{\{1\}, \{2\} \text{ ----- } \{1,2,3\}, \emptyset\}$.

Note :- The no. of Elements in power set is Equal to 2^n.

Proper set :- If A, B are two sets such that. $B \subseteq A$ and $B \neq A$ then A, B are. called proper sets.

Ex :- $A = \{1,2,3\}$ $B = \{3,4\}$

$B \subseteq A$ & $B \neq A$.

$\therefore$ A, B are proper sets.

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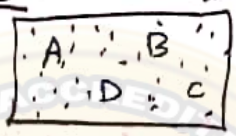
Note :- $n(A) = n$, then no. of Proper Sets Equal to $2^n - 1$

Super Set :- Let A, B are two Sets and $A \subseteq B$, $B \subseteq A$. then A is called Super Set of B.

Universal Set :- Any Set which including all the various Sets is called universal Set. and it is denoted by 'E' (or) 'u' or 'U'.

Symbol of universal Set :-



Ex :- $u = \{$ Set of real numbers $\}$; $A = \{$ Set of real numbers $\mathbb{R}\}$

$B = \{$ Set of real Neg. numbers $\}$; $C = \{$ Set of real numbers Pg. Numbers $\}$.

$\therefore A \subseteq u$, $B \subseteq u$, $C \subseteq u$,

→ $A = \{1, 2, 3\} \in N, W, R, \mathbb{Z}$ or.

$B = \{-1, -2, -3\}$

$C = \{\frac{1}{2}, \frac{1}{3}, \frac{1}{4}\}$

$\therefore u =$ or. (or) R.

Set operations :- 5th Nov

→ **union :-** Let A, B are two Sets the union of two Sets A, B is ~~Sets~~ denoted by $A \cup B$. The union of $A \cup B$ is contains those elements either A (or) B.

$A \cup B = \{x / x \in A$ or $x \in B\}$

$= \{x / x \in A \vee x \in B\}$

Ex :- $A = \{1, 2, 3\}$, $B = \{1, 2\}$

$A \cup B = \{1, 2, 3\}$

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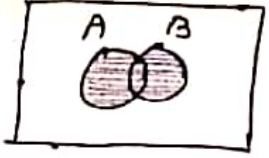
VENN diagram :—



$A \cup B =$

Intersection :— Let A, B are two sets the intersection of two sets is denoted by $A \cap B$ the intersection contains elements both A & B.

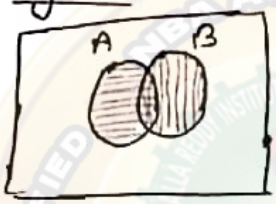
$$A \cap B = \{x / x \in A \text{ and } x \in B\}$$
$$= \{x / x \in A \wedge x \in B\}$$

(33)

Ex:— $A = \{1, 2, 3\}$, $B = \{2, 3\}$

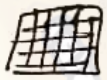
$A \cap B = \{2, 3\}$.

VENN diagram :—



$A =$

$B =$

$A \cap B =$ 

5th NOV

Disjoint sets :— Let A, B are two sets and $A \cap B = \phi$ then A, B are called disjoint sets

Ex:— $A = \{1, 2, 3\}$ $B = \{4, 5, 6\}$

$A \cap B = \phi$

Compliment of a Set :— Let u is a universal set and A be a set then $u - A$ is called compliment of A. and it is denoted by A'. (or) A^c.

$\therefore A' = A^c = u - A$.

Ex:— $A = \{1, 2, 3, 4, 5\}$

$B = \{1, 2, 3\}$

$A' = u - A = \{4, 5\}$

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VENN Diagram:-

A' =

$A \cap B = \{x / x \in A \text{ and } x \in B\}$

$A \cup B = \{x / x \in A \text{ or } x \in B\}$

$A' = U - A = \{x \in A / x \notin A'\}$

$= \{x \in A / x \notin U - A\}$

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Difference b/w two sets :- Let A, B are two sets then A-B is called Difference of A, B.

Ex:- $A = \{1, 2, 3\}$ $B = \{1, 2\}$

$A - B = \{3\}$

$\Rightarrow A - B = \{x / x \in A - B\}$

$= \{x / x \in A \text{ and } x \notin B\}$

$= \{x / x \in A \text{ and } x \in B'\}$

$= \{x / x \in A \cap B'\}$

Note :- 1. If A, A' are two sets then $A \cap A' = \phi$ and $A \cup A' = U$

2. $A - B \neq B - A$.

* Symmetric of two sets (or) symmetric difference of two sets * :-

Let A, B are two sets then $(A-B) \cup (B-A)$ is called symmetric difference of A, B. It is denoted by $A \Delta B$.

$\therefore A \Delta B = (A - B) \cup (B - A)$ (or)

$A \Delta B = (A \cap B) - (B \cup A)$

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Properties of union & Intersection :—

closure law :— Let A, B are two sets. (i) $A \cap B$ (ii) $A \cup B$.

Associative law :— 1. $(A \cap B) \cap C = A \cap (B \cap C)$

2. $(A \cup B) \cup C = A \cup (B \cup C)$

Identity law :— 1. Let A be a set such that $A \cup \phi = \phi \cup A = A$ then ϕ is called identity set of union.

2. Let A be a set such that $A \cap U = U \cap A = A$, then U is called identity of intersection.

Commutitive law :— Let A, B are two sets

1. $A \cup B = B \cup A$.
2. $A \cap B = B \cap A$.

Distributive law :— 1. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

2. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Demorgance Law :— Let A, B are two sets then.

1. $(A \cup B)' = A' \cap B'$.
2. $(A \cap B)' = A' \cup B'$.

→ $(A')' = A$., $A \cup A = A$, $A \cap A = A$.

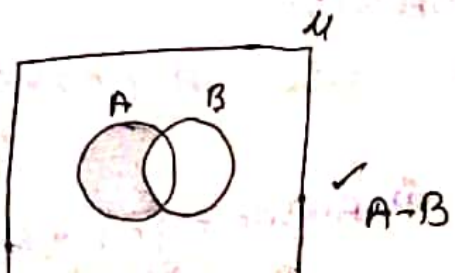
→ $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

→ $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$

→ $n(A \Delta B) = n(A) + n(B) - 2n(A \cap B)$

→ $n(A' \cup B') = n(U) - n(A \cap B)$

→ VENN diagram of A-B :—



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Problems:—

1) If $A=\{2,3,4\}$, $B=\{1,2\}$, $C=\{4,5,6\}$, find (i) $A \cup B$, (ii) $B \cup C$ (iii) $C \cup A$. (iv) $(A \cup B \cup C)$ (v) $A \cap B$, $B \cap C$, $C \cap A$, (vi) $A \cap B \cap C$, $n(A \cup B \cup C)$

Sol:- Given $A=\{2,3,4\}$ $B=\{1,2\}$ $C=\{4,5,6\}$

1. $A \cup B = \{1,2,3,4\}$
2. $B \cup C = \{1,2,4,5,6\}$
3. $C \cup A = \{2,3,4,5,6\}$
4. $A \cup B \cup C = \{1,2,3,4,5,6\}$
5. $A \cap B = \{2\}$
   $B \cap C = \{\}$
   $C \cap A = \{4\}$
6. $A \cap B \cap C = \{\}$
   $n(A \cup B \cup C) = 6$ /-

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2. Let $A=\{1,2,3,4,5\}$ $B=\{0,3,6\}$ then find (i) $A \cup B$ (ii) $A \cap B$ (iii) $A-B$ (iv) $B-A$.

Sol:- Given $A=\{1,2,3,4,5\}$ $B=\{0,3,6\}$

(i) $A \cup B = \{0,1,2,3,4,5,6\}$

(ii) $A \cap B = \{3\}$

(iii) $A-B = \{1,2,4,5\}$

(iv) $B-A = \{0,6\}$ /-

3) Let $A=\{0,2,4,6,8,10\}$ $B=\{0,1,2,3,4,5,6\}$ $C=\{4,5,6,7,8,10\}$ then find (i) $A \cap B \cap C$ (ii) $A \cup B \cup C$ (iii) $(A \cup B) \cap C$ (iv) $(A \cap B) \cup C$

Sol:- Given $A=\{0,2,4,6,8,10\}$, $B=\{0,1,2,3,4,5,6\}$

$C=\{4,5,6,7,8,10\}$

(i) $A \cap B \cap C = \{0,2,4,6,8,10\} \cap \{0,1,2,3,4,5,6\} \cap \{4,5,6,7,8,10\}$

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(ii) $A \cup B \cup C = \{0,1,2,3,4,5,6,7,8,10\}$

(iii) $(A \cup B) \cap C = \{0,1,2,3,4,5,6,7,8,10\} \cap \{4,5,6,7,8,10\}$

$= \{4,5,6,8,10\}$

(iv) $(A \cap B) \cup C = \{2,4,6\} \cup \{4,5,6,7,8,10\}$

$= \{2,4,5,6,7,8,10\}$ //

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4) If $A = \{a,b,c,d,e\}$ $B = \{a,b,c,d,e,f,g,h\}$ then find

(i) $A \cup B$ (ii) $A \cap B$ (iii) $A - B$ (iv) $B - A$.

Sol:- $A \cup B = \{a,b,c,d,e,f,g,h\}$

$A \cap B = \{a,b,c,d,e\}$

$A - B = \{\}$

$B - A = \{f,g,h\}$ //

5) If $A = \{2,3,4\}$ $B = \{1,2\}$ $C = \{4,5,6\}$ then find ~~A (i) (A-B), (B~~

(i) $(A-B), (B-A)$ (ii) $B-C, C-B$ (iii) $A-(B+C), (B+C)-A$.

(iv) $A+B$ (v) $B+C$, (vi) $A+B+C$ (vii) $(A+B)+(B+C)$

Sol:- (i) $A - B \Rightarrow \{2,3,4\} - \{1,2\}$

$= \{2\} = \{3,4\}$

$B - A \Rightarrow \{1,2\} - \{2,3,4\} = \{1\}$

(ii) $B-C$, $C-B$

$B - C = \{1,2\} - \{4,5,6\}$; $C - B = \{\}$

$= \{\}$

(iii) $A - (B+C) = \{2,3,4\} - \{1,2,4,5,6\}$

$= \{3\}$

$(B+C) - A = \{1,2,4,5,6\} - \{2,3,4\} = \{1,5,6\}$

(iv) $A+B = (A-B) \cup (B-A)$

$= \{3,4\} \cup \{1\}$

$= \{1,3,4\}$

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(v) $B+C = (B-C) \cup (C-B)$

$= \{1,2\} \cup \{4,5,6\}$

$= \{1,2,4,5,6\}$

(vi) $A+B+C = A+(B+C)$

$= \{A-(B+C)\} \cup [(B+C)-A]$

$= \{3\} \cup \{1,5,6\}$

$= \{1,3,5,6\}$

(vii) $(A+B)+(B+C) = [(A+B)-(B+C)] \cup [(B+C)-(A+B)]$

$= [\{1,2,3,4\} - \{1,2,4,5,6\}] \cup [\{1,2,4,5,6\} - \{1,3,4\}]$

$= [\{3\} \cup \{2,5,6\}]$

$(A+B)+(B+C) = \{2,3,5,6\}$ /—

6) P.T $(\overline{A \cap B}) = \bar{A} \cup \bar{B}$ (or) $(A \cap B)^c = A^c \cup B^c$ by using truth table.

Sol:—

A	B	$\bar{A}$	$\bar{B}$	$A \cap B$	$\overline{A \cap B}$	$\bar{A} \cup \bar{B}$
1	1	0	0	1	0	0
1	0	0	1	0	1	1
0	1	1	0	0	1	1
0	0	1	1	0	1	1

$\cup - \vee$

$\cap - \wedge$

$\therefore (A \cap B)^c = A^c \cup B^c$ /—

7) P.T $(\overline{A \cap B}) = \bar{A} \cup \bar{B}$ by using builder notation.

Sol:— L.H.S $(\overline{A \cap B}) = \{x / x \in (\overline{A \cap B})\}$

$= \{x / x \notin A \cap B\}$

$= \{x / x \notin A \text{ and } x \notin B\}$

$= \{x / x \notin A \cup x \notin B\}$

$= \{x / x \notin \bar{A} \cup x \in \bar{B}\}$

$= \{x / x \in \bar{A} \cup \bar{B}\}$

$= R.H.S$

$\therefore \overline{A \cap B} = \bar{A} \cup \bar{B}$ /—

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8. P.T. Associative law in union and intersection by using tabular form.

Sol:-

A	B	C	A∪B	(A∪B)∪C	B∪C	A∪(B∪C)
1	1	1	1	1	1	1
1	1	0	1	1	1	1
1	0	1	1	1	1	1
1	0	0	1	1	0	1
0	1	1	1	1	1	1
0	1	0	1	1	1	1
0	0	1	0	1	1	1
0	0	0	0	0	0	0

∴ (A∪B)∪C = A∪(B∪C) /—

A	B	C	A∩B	(A∩B)∩C	B∩C	A∩(B∩C)
1	1	1	1	1	1	1
1	1	0	1	0	0	0
1	0	1	0	0	0	0
1	0	0	0	0	0	0
0	1	1	0	0	1	0
0	1	0	0	0	0	0
0	0	1	0	0	0	0
0	0	0	0	0	0	0

∴ (A∩B)∩C = A∩(B∩C) /—

9) P.T. (A∪B)∪C = A∪(B∪C) by using builder notation.

Sol:- L.H.S $(A\cup B)\cup C = \{x / x \in (A\cup B)\cup C\}$

$= \{x / x \in A\cup B$ (or) $x \in C\}$

$= \{x / x \in A$ or $x \in B$ or $x \in C\}$

$= \{x / x \in A$ or $(x \in B$ or $x \in C)\}$

$= \{x / x \in A$ or $(x \in B\cup C)\}$

$= \{x / x \in A \cup x \in B\cup C\}$

$= \{x / x \in A\cup(B\cup C)\}$

RHS /—

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10. Prove that distributive laws by using table form & building form.

Sol:-

A	B	C	$(A \cup B)$	$(A \cup B) \cap C$	$A \cap C$	$B \cap C$	$(A \cap C) \cup (B \cap C)$
1	1	1	1	1	1	1	1
1	1	0	1	0	0	0	0
1	0	1	1	1	1	0	1
1	0	0	1	0	0	0	0
0	1	1	1	1	0	1	1
0	1	0	1	0	0	0	0
0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0

$\therefore (A \cup B) \cap C = (A \cap C) \cup (B \cap C)$ (40)

10th NOV

LHS.

$(A \cup B) \cap C = \{x / x \in (A \cup B) \cap C\}$

$= \{x / x \in A \cup B \text{ and } x \in C\}$

$= \{x / (x \in A \text{ or } x \in B) \text{ and } x \in C\}$

$= \{x / (x \in A \text{ and } x \in C) \text{ or } (x \in B \text{ and } x \in C)\}$

$= \{x / (x \in A \cap C) \text{ or } (x \in B \cap C)\}$

$= \{x / (x \in A \cap C) \cup (x \in B \cap C)\}$

$= \{x / x \in (A \cap C) \cup (B \cap C)\}$ /—

* family of union:—

If $A_1, A_2, A_3 \ldots\ldots A_n$ are sets then $A_1 \cup A_2 \cup \ldots\ldots A_n$ is called family of union set and it is denoted by $\bigcup_{i=1}^{n} A_i$

* family of intersection:—

If $A_1, A_2, A_3 \ldots\ldots A_n$ are sets. then $A_1 \cap A_2 \cap \ldots\ldots A_n$ is called family of intersection and it is denoted by $\bigcap_{i=1}^{n} A_i$

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Problems:-

1) verify $B \cup (\bigcap_{i \in j} A_i) = \bigcap_{i \in j} (B \cup A_i)$ and

$B \cap (\bigcup_{i \in j} A_i) = \bigcup_{i \in j} (B \cap A_i)$ such that $A_1 = \{0,1,2\}$

$A_2 = \{0,3,4,5\}$, $A_3 = \{4,7,8\}$, $B = \{2,7\}$ $j = \{1,2,3\}$

Sol:- (i) First we have to S.T $B \cup (\bigcap_{i \in j} A_i) = \bigcap_{i \in j} (B \cup A_i)$

L.H.S $B \cup (\bigcap_{i \in j} A_i)$

$$\bigcap_{i \in j} A_i = \bigcap_{i=1}^{3} A_i$$

$$= A_1 \cap A_2 \cap A_3$$

$$= \{0,1,2\} \cap \{0,3,4,5\} \cap \{4,7,8\}$$

$$= \{0\} \cap \{4,7,8\}$$

$$= \{\}$$

$$B \cup (\bigcap_{i \in j} A_i) = \{2,7\} \cup \{\}$$

$$= \{2,7\}$$

R.H.S.

$$\bigcap_{i \in j} (B \cup A_i) = \bigcap_{i=1}^{3} (B \cup A_i)$$

$$= (B \cup A_1) \cap (B \cup A_2) \cap (B \cup A_3)$$

$$= \{0,1,2,7\} \cap \{0,2,3,4,5,7\} \cap \{2,4,7,8\}$$

$$= \{0,2,7\} \cap \{2,4,7,8\}$$

$$= \{2,7\}$$

$\therefore$ L.H.S = R.H.S.

(ii) First we have to S.T $B \cap (\bigcup_{i \in j} A_i) = \bigcup_{i \in j} (B \cap A_i)$

L.H.S. $B \cap (\bigcup_{i \in j} A_i)$

$$\bigcup_{i \in j} A_i = \bigcup_{i=1}^{3} A_i$$

$$= A_1 \cup A_2 \cup A_3$$

$$= \{0,1,2,3,4,5,7,8\}$$

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$B \cap (\bigcup_{i \in J} A_i)$

$\{2,7\} \cap \{0,1,2,3,4,5,7,8\}$

$= \{2,7\}$.

R.H.S.

$\bigcup_{i \in J} (B \cap A_i) = \bigcup_{i=1}^{3} (B \cap A_i)$

$= (B \cap A_1) \cup (B \cap A_2) \cup (B \cap A_3)$

$= \{2\} \cup \{\} \cup \{7\}$

$= \{2,7\}$

$\therefore$ LHS = RHS /—

—

Functions :—

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functions

Pair of Orders :— Two Elements are arranged in open bracket with comma's. then it is called Pair of Order.

If a,b are two Elements then (a,b) are called Pair of Order.

Note:— 1. In (a,b) a is called first Element and 'b' is called second Element.

2. $(a,b) \neq (b,a)$

3. $(a,b) \neq \{a,b\}$

4. $(a,b) = (x,y)$ then $a=x$ & $b=y$.

Cartision Product of A & B :—

The set of pair of Orders is called Cartision Product.

(B1)

If A & B are two sets then the set $\{(a,b) / a \in A, b \in B\}$ is called Cartision Product of A & B and it is denoted by 'A × B'.

$\therefore A \times B = \{(a,b) / a \in A, b \in B\}$

Ex:— $A = \{1,2,3,4\}$ $B = \{a,b,c\}$

$A \times B = \{(1,a)(1,b)(1,c)(2,a)(2,b)(2,c)$
$(3,a)(3,b)(3,c)(4,a)(4,b)(4,c)\}$

Ily $B \times A = \{(a,1)(a,2)(a,3)(a,4)(b,1)(b,2)(b,3)(b,4)$
$(c,1)(c,2)(c,3)(c,4)\}$

Note:— 1. $A \times B \neq B \times A$

2. $n(A \times B) = n(B \times A)$

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Relation :— The Subset of Cartisian Product is called relation. and it is denoted by 'R'.

∴ Relation 'R' = $\{(a,b) / (a,b) \in A \times B\}$

Problem :—

1) write all the Elements of $\{a,b\} \times \{1,2,3\}$

Sol:— $A = \{a,b\}$ $B = \{1,2,3\}$

$A \times B = \{(a,1)(a,2)(a,3)$
$(b,1)(b,2)(b,3)\}$

2) $A = \{1\}$ $B = \{a,b\}$ $C = \{2,3\}$ then find

i. $A \times B \times C$ (ii) A^3 (iii) B^2 (iv) $B^2 \times C$ (v) $A \times B$.

Sol:— Given $A = \{1\}$, $B = \{a,b\}$; $C = \{2,3\}$

1. $A \times B \times C$ = Not defined.

2. $A^3 = A \times A = \{1\} \times \{1\}$
$= \{(1,1)\}$
$A^2 \times A = \{1,1\} \times \{1\}$
$A^3 = \{(1,1),(1,1)\}$

3. $B^2 = \{a,b\} \times \{a,b\}$
$= \{(a,a)(a,b)$
$(b,a)(b,b)\}$

4. $B^2 \times C$ = Not defined

5. $A \times B = \{1\} \times \{a,b\}$
$= \{(1,a),(1,b)\}$ /—

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Function :-

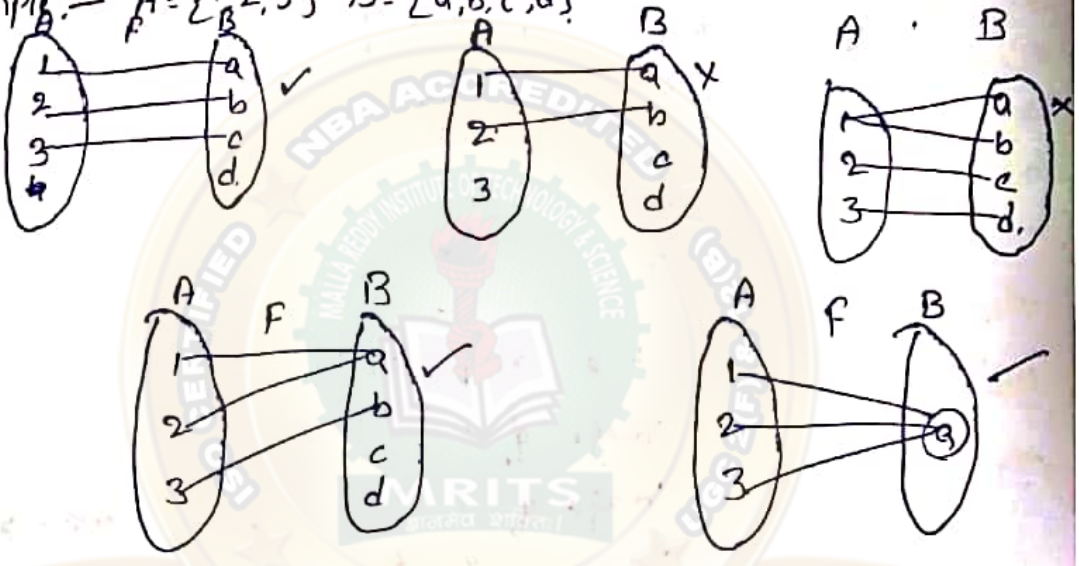
12th Nov

Let A & B are two sets if every element in A mapping to B. Set unique time (or) one time. then the mapping is called function and it is denoted $f : A \to B$.

(45)

→ In a function $F : A \to B$ 'A' is called domain and 'B' is called co-domain.

Example :- $A = \{1, 2, 3\}$ $B = \{a, b, c, d\}$



Range :- The set of image element is called Range. and it is denoted by $f(a)$

$\therefore$ Range $= F(a)$

$F : A \to B$

Range $= \{a \in A / f(a) = a'\}$ (B)

$\{a \in A / f(a) \in B\}$

Domain $(A) = \{1, 2, 3\}$

Co-Domain $(B) = \{a, b, c, d\}$

Range $= \{a, b\}$

Note :- 1. Range is subset to co-domain that is $f(a) \subseteq B$.

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Types of functions :—

1. one-one function (or) Injection :—

A function is called one-one function. If different Elements in A' having different Images in B.

Note :— Let $F : A \to B$ is a function

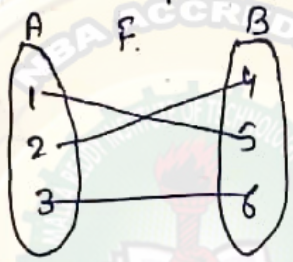
$F : A \to B$

$a_1, a_2 \in A$ (Elements)

$f(a_1), f(a_2) \in B$ (Image)

If $a_1 = a_2 \Rightarrow f(a_1) = f(a_2)$

If $a_1 \neq a_2 \Rightarrow f(a_1) \neq f(a_2)$



∵ F is a function.

Image of 1 = 5

Image of 2 = 4

Image of 3 = 6.

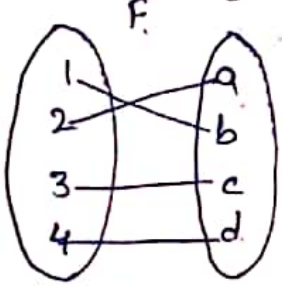
∴ F is one-one.

→ onto function or Surjection :— A function $f : A \to B$ is called onto function. If its Co-Domain = Range.

Note :— $F : A \to B$ is a onto function.

If their Exist $a \in A$ such that $f(a) = b$, $b \in B$.

Ex. :— 1.



$A = \{1, 2, 3, 4\}$

$B = \{a, b, c, d\}$

Range $= \{a, b, c, d\}$

B = Range

∴ F is onto function.

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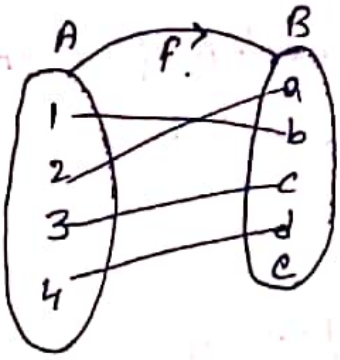
11.



$A = \{1, 2, 3, 4\}$

$B = \{a, b, c, d, e\}$

$B \neq$ Range. Range $= \{a, b, c, d\}$

∴ f is not onto.

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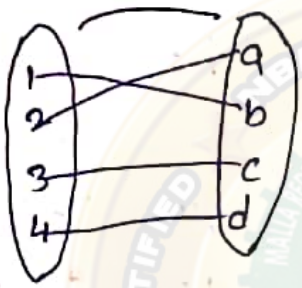
Bijection :— A function $f: A \to B$ is called Bijection. If it ~~conje~~ contains one-one and ~~one to~~ onto.

Ex :—



$A = \{1, 2, 3, 4\}$

$B = \{a, b, c, d\}$

Range $= \{a, b, c, d\}$

$B =$ Range.

∴ ~~F is~~ ~~fun~~

If $1 = b$

$2 = a$.

$3 = c$

$4 = d$

∴ f is 1-1

∴ f is onto

∴ f is Bijection.

13th Nov

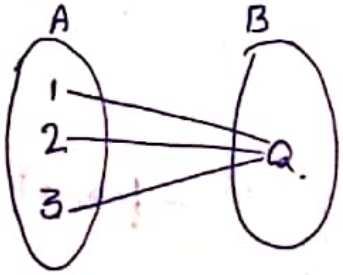
Constant function :—

A function. $f: A \to B$ is called constant function. if Every element in 'A' mapping to 'B' constant element.

(or)

A function $f: A \to B$ is said to be constant function if $f(a) = c$ (constant).

Ex :—



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Identity function :— A function $f: A \to A$ is called identity function. If $f(a) = a$, $a \in A$.

Problems :—

1) Let $f(x) = \frac{x^2}{3}$ then find $f(S)$, If

i. $S = \{-2, -1, 0, 1, 2, 3\}$ (ii) $S = \{0, 1, 2, 3, 4, 5\}$ (48)

(iii) $S = \{1, 5, 7, 11\}$ (iv) $S = \{2, 6, 10, 14\}$

Sol:— i. $S = \{-2, -1, 0, 1, 2, 3\}$

$f(x) = \frac{x^2}{3}$

$f(-2) = 4/3$

$f(-1) = 1/3$

$f(0) = 0$

$f(1) = 1/3$

$f(2) = 4/3$

$f(3) = 9/3 = 3$.

(ii) $S = \{0, 1, 2, 3, 4, 5\}$

$f(0) = 0$

$f(1) = 1/3$

$f(2) = 4/3$

$f(3) = 9/3 = 3$

$f(4) = 16/3$

$f(5) = 25/3$.

(iii) $S = \{1, 5, 7, 11\}$

$f(1) = 1/3$

$f(5) = 25/3$

$f(7) = 49/3$

$f(11) = \frac{121}{3}$

(iv) $S = \{2, 6, 10, 14\}$

$f(2) = 4/3$

$f(6) = \frac{36}{3}$

$f(10) = \frac{100}{3}$

$f(14) = \frac{196}{3}$ /—

2) Let $S = \{-1, 0, 2, 4, 7\}$ find $f(S)$ If.

i. $f(x) = 1$

Sol:— Given that $S = \{-1, 0, 2, 4, 7\}$ & $f(x) = 1$.

$f(-1) = 1$

$f(0) = 1$

$f(2) = 1$

$f(4) = 1$

$f(7) = 1$ /—

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called

3) If $S=\{-1,0,2,4,7\}$ find $f(S)$ if $f(x)=2x+1$.

Sol:- Given $S=\{-1,0,2,4,7\}$ & $f(x)=2x+1$.

$f(-1)=-1$ $\quad$ $f(4)=9$

$f(0)=1$ $\quad$ $f(7)=15$ /-

$f(2)=5$

(49)

4) Let $S=\{-1,0,2,4,7\}$ find $f(S)$ if $f(x)=\frac{x^2+1}{3}$.

Sol:- Given $S=\{-1,0,2,4,7\}$ & $f(x)=\frac{x^2+1}{3}$.

$f(-1)=\frac{(-1)^2+1}{3}=\frac{2}{3}$ $\quad$ $f(4)=\frac{(4)^2+1}{3}=\frac{17}{3}$

$f(0)=\frac{0+1}{3}=1/3$ $\quad$ $f(7)=\frac{(7)^2+1}{3}=\frac{50}{3}$ /-

$f(2)=\frac{(2)^2+1}{3}=5/3$

Inverse function :- A function $f: A\to B$ is bijective then f^{-1} exists.

If $f: A\to B$ is a function. then $B\to A$ is called inverse function of f: and it is denoted by $f^{-1}: B\to A$

Note:- If $a\in A$ then $f^{-1}(b)=a$. ⇔

If $x\in A$ then. $f^{-1}(y)=x$.

Problems:-

1) If f is a function from $A\to R$ defined by $f(x)=x^2, x\in A$. verify f is one-one, onto, & Bijection. If it is a bijection then find inverse of f:

14th NOV

Sol:- Given such that $f: R\to R$.

$f(x)=x^2$.

$A=R=\{0,1,-1,2,-2,\text{-----}\}$

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$f(x) = x^2$

$f(0) = 0 = (0, 0)$

$f(1) = 1 = (1, 1)$

$f(-1) = 1 = (-1, 1)$

$f(2) = 4 = (2, 4)$

$f(-2) = 4 = (-2, 4)$

∵ f is a function

∴ f is one-one

∴ f is onto

∴ f is not bijection.

Inverse does not exist.

2) A function from $\{a, b, c, d\}$ to $\{1, 2, 3, 4, 5\}$ with $f(a) = 4$, $f(b) = 5$, $f(c) = 1$, $f(d) = 3$. then show, that it is a one-one function.

Sol:- Given function from $\{a, b, c, d\}$ to $\{1, 2, 3, 4, 5\}$

$f(a) = 4$, $f(b) = 5$, $f(c) = 1$ & $f(d) = 3$

Domain $(A) = \{a, b, c, d\}$

Co-domain $(B) = \{1, 2, 3, 4, 5\}$

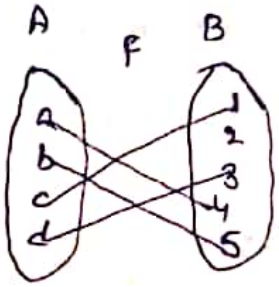

Image of $a = 4$.

Image of $b = 5$

Image of $c = 1$

Image of $d = 3$

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∴ different Elements having different images

∴ f is one-one /-

3. P.T A function $f(x) = x^2$ from the Set of Integers to Set of Integers it is not a one-one function.

Sol:- Given $f(x) = x^2$

Define. $f : z \to z$

$z = \{-, 0, +\}$

Domain $= z = \{-, 0, +\}$

$f(x) = x^2$.

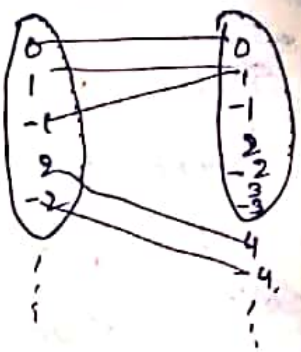
$f(0) = 0$, $(0,0)$

$f(1) = 1$, $(1,1)$

$f(-1) = 1$, $(-1,1)$

$f(2) = 4$, $(2,4)$

$f(-2) = 4$, $(-2,4)$



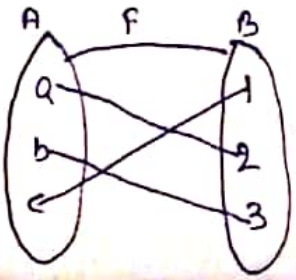
∴ f is Note one-one Because Image of 1 is & Image of -1 is also 1 /-

4) Let F is the function from Set of $\{a,b,c\}$ to $\{1,2,3\}$ such that $f(a)=2$, $f(b)=3$ & $f(c)=1$ then verify F is a Inverse. and. If it is Inverse then find F^{-1}.

Sol:- Given Function $\{a,b,c\}$ to $\{1,2,3\}$

$f(a)=2$, $f(b)=3$, $f(c)=1$.

$A = \{a,b,c\}$ $B = \{1,2,3\}$



∴ F is function

∴ F is 1-1

∴ F is onto

∴ F is 1-1 and onto

∴ F is bijection

∴ F^{-1} is Exists.

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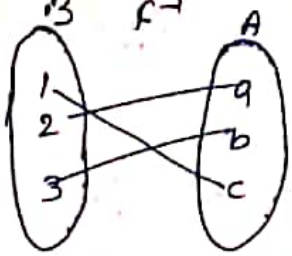
Find (f^{-1})



$f(a) = 2$
$f(b) = 3$
$f(c) = 1$

$f^{-1}(2) = a$
$f^{-1}(3) = b$
$f^{-1}(1) = c$ //

** 5) Let $f: z \to z$ be such that $f(x) = x+1$. then show that f is one one, onto, bijection & inverse and also find $f^{-1}(x)$

Sol:-

Given function $f: z \to z$. is $f(x) = x+1$.

Domain(z) = $\{-, 0, +\}$

$f(x) = x+1$

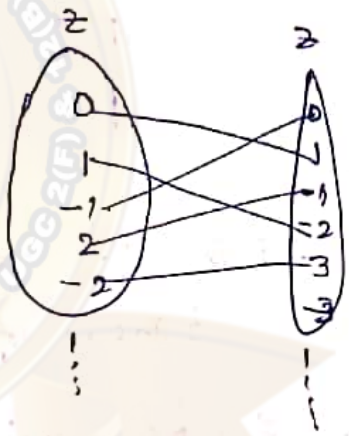
$f(0) = 1 \Rightarrow (0, 1)$

$f(1) = 2 \Rightarrow (1, 2)$

$f(-1) = 0 \Rightarrow (-1, 0)$

$f(2) = 3 \Rightarrow (2, 3)$

$f(-2) = -1 \Rightarrow (-2, -1)$



∴ f is function

∴ f is 1-1

∴ f is onto

∴ f is 1-1 & onto

∴ f is bijective.

∴ f^{-1} Exist.

find $f^{-1}(x)$

$f(x) = x+1$.

$f(x) = y \Rightarrow x = f^{-1}(y)$

$y = x+1$

$x = y-1$

∴ $f^{-1}(y) = x$.

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$f^{-1}(y) = y-1$

(8)

$f^{-1}(x) = x-1$. //—

6) A function $f: R \to R$ is defined by $f(x) = x^3 - 2$ then find $f^{-1}(x)$

Sol:— $f(x) = x^3 - 2$

Let $f(x) = y$

$x^3 - 2 = y$.

$x^3 = y + 2$

$x = (y+2)^{1/3}$

$f^{-1}(y) = x$

$f^{-1}(y) = (y+2)^{1/3}$

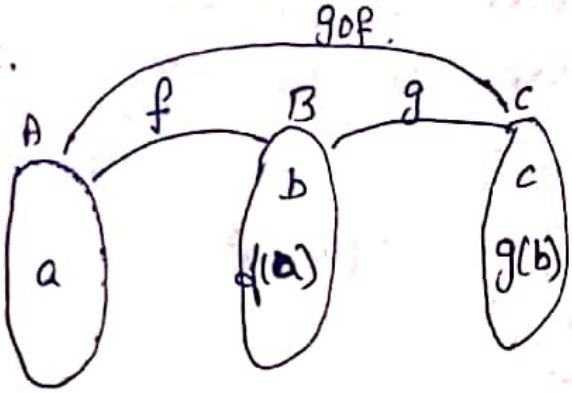
(8)

$f^{-1}(x) = (x+2)^{1/3}$ //—

15th Nov

Composite numbers (8) composition function:—

Let A, B, C are three non-empty sets such that. $f: A \to B$, $g: B \to C$ are two functions then $A \to C$ is called composition function, and it is denoted by 'gof' from $A \to C$.



Note:— 1. $fog(x) = f[g(x)]$

2. $fog \neq gof$

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Problems:-

1) Let f, g are two functions from the set of Integers to set of integers defined by $f(x) = 2x+3$, & $g(x) = 3x+2$. what is the composition of $f \& g$, $g \& f$.

Sol:- Given $f: Z \to Z$ & $g: Z \to Z$.

$f(x) = 2x+3$

$g(x) = 3x+2$.

i. $fog(x) = f[g(x)]$

$= f[3x+2]$ $\quad f(x) = 2x+3$

$= 2(3x+2)+3$

$= 6x+4+3$

$= 6x+7$.

ii. $gof(x) = g[f(x)]$

$= g[2x+3]$ $\quad g(x) = 3x+2$.

$= 3[2x+3)+2$

$= 6x+9+2$

$= 6x+11$

$\therefore fog \neq gof$ /-

2) $F: R \to R$. is defined by $f(x) = x^2$ & $g: R \to R$ is defined by $g(x) = \frac{x}{x-2}$ find fog & gof.

Sol:- Given $F: R \to R$. is $f(x) = x^2$

$g: R \to R$ is $g(x) = \frac{x}{x-2}$.

i. $fog(x) = f[g(x)]$

$= f\left[\frac{x}{x-2}\right]$ $\quad \because f(x) = x^2$.

$= \left(\frac{x}{x-2}\right)^2$

$= \frac{x^2}{x^2+4-4x}$

ii) $gof(x) = g[f(x)]$

$= g[x^2]$

$= \frac{x^2}{x^2-2}$ /-

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3. Let 'g' be the function from the set {A, B, c} to itself. Such that g(a) = b, g(b) = c, g(c) = a. Let 'f' be the function from {A, B c} to the set {1, 2, 3} such that f(a) = 3, f(b) = 2 f(c) = 1. what is the composition of f and g, and what is the composition of g & f.

Sol:- Given that $g: \{a, b, c\} \to$ itself.

$g(a) = b \Rightarrow (a, b)$

$g(b) = c \Rightarrow (b, c)$

$g(c) = a \Rightarrow (c, a)$

$f: \{a, b c\} \to \{1, 2, 3\}$

$f(a) = 3 \Rightarrow (a, 3)$

$f(b) = 2 \Rightarrow (b, 2)$

$f(c) = 1 \Rightarrow (c, 1)$

(i) fog

$g = \{a, b, c\}$

$fog(a) = F[g(a)]$

$= f(b)$

$= 2 \Rightarrow (a, 2)$

$fog(b) = f[g(b)]$

$= f(c)$

$= 1 \Rightarrow (b, 1)$

$fog(c) = f[g(c)]$

$= f(a)$

$= 3 \Rightarrow (c, 3)$

$\therefore fog = \{(a, 2), (b, 1), (c, 3)\}$.

(ii). gof.

$f = \{a, b, c\}$

$gof(a) = g[f(a)]$

$= g(3)$

not defined.

$\therefore$ gof not defined //-

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Mathemetical Induction :- Let $P(n)$ be a Statement then

1. To P.T $P(n)$ is true for $n=1$.
2. Let us suppose that $P(n)$ is true for $n=k$.
3. To P.T $P(n)$ is true for $n=k+1$.

$\therefore$ $P(n)$ is true for all $n \in N$.

(56)

Problems:-

1) To P.T $1+2+3+\dots+n = \frac{n(n+1)}{2}$ by using to mathematical Induction.

Sol:- $P(n) = 1+2+3+\dots+n = \frac{n(n+1)}{2} \rightarrow (1)$

If $n=1$.

$n = \frac{n(n+1)}{2}$

$1 = \frac{1(1+1)}{2}$

$1 = 2/2$

$1 = 1$.

LHS = RHS.

$\therefore P(n)$ is true for $n=1$.

Let us suppose that $P(n)$ is true for $n=k$.

$1+2+3+\dots+k = \frac{k(k+1)}{2} \rightarrow (2)$

If $n=k+1$,

n^{th} term $= n$

$(k+1)^{th}$ term $= (k+1)$

$(k+1)^{th}$ term Adding b.s Eqn (2)

$1+2+3+\dots+k+(k+1) = \frac{k(k+1)}{2} + (k+1)$

$= (k+1)\left[\frac{k}{2}+1\right]$

$= \frac{(k+1)(k+2)}{2}$

$= \frac{(k+1)((k+1)+1)}{2}$

$\therefore P(n)$ is true for $n=k+1$ /-

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18th Nov

2) P.T $\Sigma n^2 = \frac{n(n+1)(2n+1)}{6}$ (or) $1^2+2^2+3^2+\ldots+n^2 = \frac{n(n+1)(2n+1)}{6}$

by using M.I.

Sol:- $P(n) = 1^2+2^2+3^2\ldots+n^2 = \frac{n(n+1)(2n+1)}{6} \rightarrow (1)$

If $n=1$.

$n^2 = \frac{n(n+1)(2n+1)}{6}$

$1^2 = \frac{1(1+1)(2+1)}{6}$

$1 = \frac{2\times3}{6} \Rightarrow 1 = \frac{6}{6} \Rightarrow 1=1$

$\therefore$ LHS = RHS.

$\therefore$ P(n) is true for $n=1$

Let us suppose that P(n) is true for $n=k$.

$1^2+2^2+\ldots k^2 = \frac{k(k+1)(2k+n)}{6} \rightarrow (2)$

If $n=k+1$

n^{th} term $= n^2$

$(k+1)^{th}$ term $= (k+1)^2$

Adding $(k+1)^{th}$ term B.S in Eqn (2)

$1^2+2^2+\ldots+k^2+(k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2$

$= (k+1)\left[\frac{k(2k+1)}{6} + k+1\right]$

$= (k+1)\left[\frac{2k^2+k+6k+6}{6}\right]$

$= (k+1)\left[\frac{2k^2+7k+6}{6}\right]$

$= \frac{k+1}{6}\left[2k^2+7k+6\right]$

$= \frac{k+1}{6}\left[2k^2+3k+4k+6\right]$

$= \frac{k+1}{6}\left[2k(k+2)+3(k+2)\right]$

$= \frac{(k+1)}{6}(2k+3)(k+2)$

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$$= \frac{(k+1)(k+1+1)(2k+2+1)}{6}$$

$$= \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$$ ⊢.

∴ P(n) is true for $n = k+1$.

∴ P(n) is true for $n \in N$. ⊢ (58)

3. P.T $\Sigma n^3 = \frac{n^2(n+1)^2}{4}$ by using M.I. (8)

P.T $1^3+2^3+3^3+\text{-----}+n^3 = \frac{n^2(n+1)^2}{4}$.

Sol:- $P(n) = 1^3+2^3+3^3+\text{-----}+n^3 = \frac{n^2(n+1)^2}{4}$ → (1)

If $n=1$,

$n^3 = \frac{n^2(n+1)^2}{4}$

$1^3 = \frac{1^2(1+1)^2}{4}$

$1 = \frac{2^2}{4}$

$1 = 4/4$

$1 = 1$

∴ LHS = RHS

∴ P(n) is true for $n=1$.

Let us suppose that P(n) is true for $n=15$

$1^3+2^3+3^3+\text{-----}+k^3 = \frac{k^2(k+1)^2}{4}$ → (2)

If $n = k+1$

n^{th} term $= n^3$

$(k+1)^{th}$ term $= (k+1)^3$

$(k+1)^{th}$ term Adding R.S. in Eqn (2)

$$1^3+2^3+\text{-----}+k^3+(k+1)^3 = \frac{k^2(k+1)^2}{4} + (k+1)^3.$$

$$= (k+1)^2\left[\frac{k^2}{4} + (k+1)\right]$$

$$= (k+1)^2\left[\frac{k^2+4k+4}{4}\right]$$

$$= (k+1)^2\left[\frac{(k+2)^2}{4}\right]$$

$$= \frac{(k+1)^2[(k+1)+1]^2}{4}$$

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$\therefore$ P(n) is true for $n = k+1$.

$\therefore$ P(n) is true for all $n \in N$ /—

4) use mathematical induction to S.T $1+2+2^2+\dots+2^n = 2^{n+1}-1$ for all non-negative integers.

Sol:- $1+2+2^2+\dots+2^n = 2^{n+1}-1$, $n \in I$.

If $n=0$

$2^n = 2^{n+1}-1$

$2^0 = 2^{0+1}-1$

$1 = 2-1$

$1 = 1$

$\therefore$ P(n) is true for $n=0$.

Let us suppde that P(n) is true for $n=k$.

$1+2+2^2+\dots+2^k = 2^{k+1}-1 \rightarrow (1)$

If $n = k+1$

n^{th} term $= 2^n$

$(k+1)^{th}$ term $= 2^{k+1}$

$2^{(k+1)}$ Adding on B.S in (1)

$1+2+2^2+\dots+2^k+2^{k+1} = 2^{k+1}-1+2^{k+1}$

$= 2 \cdot 2^{k+1}-1$

$= 2^{k+1+1} - 1$

$= 2^{(k+1)+1} - 1$

$\therefore$ P(n) is true for $n = k+1$ & $n \in I$

5) P.T $1^2+3^2+5^2+\dots+(2n+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3}$ if n is a non- -ve integer.

Sol:-

If $n=0$

$(2n+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3}$

$(0+1)^2 = \frac{(0+1)(0+1)(0+3)}{3}$

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$1 = \frac{3}{3}$

$1 = 1$

$\therefore P(n)$ is true for $n = 0$.

Let us suppose that $P(n)$ is true for $n = k$.

$$1^2 + 3^2 + 5^2 + \dots + (2k+1)^2 = \frac{(k+1)(2k+1)(2k+3)}{3} \rightarrow (1)$$

If $n = k+1$

n^{th} term $= (2n+1)^2$

$$(k+1)^2 = (2(k+1)+1)^2$$
$$= (2k+2+1)^2$$
$$= (2k+3)^2.$$

Adding 3.5 $(k+1)^{th}$ term in (1)

$$1^2 + 3^2 + 5^2 + \dots + (2k+1)^2 + (2k+3)^2 = \frac{(k+1)(2k+1)(2k+3)}{3} + (2k+3)^2$$

$$= (2k+3)\left[\frac{(k+1)(2k+1)}{3} + 2k+3\right]$$

$$= \frac{2k+3}{3}\left[(k+1)(2k+1) + 6k + 9\right]$$

$$= \frac{2k+3}{3}\left[2k^2 + k + 2k + 1 + 6k + 9\right]$$

$$= \frac{2k+3}{3}\left[2k^2 + 9k + 10\right]$$

$$= \frac{2k+3}{3}\left[2k^2 + 4k + 5k + 10\right]$$

$$= \frac{2k+3}{3}\left[2k(k+2) + 5(k+2)\right]$$

$$= \frac{2k+3}{3}\left[(k+2)(2k+5)\right]$$

$$= \frac{(k+2)(2k+3)(2k+5)}{3}$$

$$= \frac{(k+1+1)(2k+2+1)(2k+2+3)}{3}$$

$$= \frac{(k+1+1)(2(k+1)+1)(2(k+1)+3)}{3}$$

$\therefore P(n)$ is true for $n = k+1$

$\therefore P(n)$ is true for all $n \in I^+$.

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6. $1.2+2.3+\cdots+n(n+1) = \frac{n(n+1)(n+2)}{3}$ for every two integer.

Sol:- $1.2+2.3+\cdots+n(n+1) = \frac{n(n+1)(n+2)}{3} \rightarrow (1)$

If $n=1$.

$$n(n+1) = \frac{n(n+1)(n+2)}{3}$$

$$2 = \frac{1(1+1)(1+2)}{3}$$

$$2 = \frac{6}{3}^{2}$$

$$2 = 2$$

$\therefore P(n)$ is true for $n=1$.

Let us suppose that $P(n)$ is true for $n=k$.

$$1.2+2.3+\cdots+k(k+1) = \frac{k(k+1)(k+2)}{3} \rightarrow (2)$$

If $n=k+1$.

n^{th} term $= k(k+1)$

$(k+1)^{th}$ term $= (k+1)(k+1+1)$

$= (k+1)(k+2)$

$(k+1)(k+2)$ Adding 3.5 in Eqn (2)

$$1.2+2.3+\cdots+k(k+1)+(k+1)(k+2) = \frac{k(k+1)(k+2)}{3} + (k+1)(k+2)$$

$$= (k+1)(k+2)\left(\frac{k+3}{3}\right)$$

$$= \frac{(k+1)((k+1)+1)((k+1)+2)}{3}$$

$\therefore P(n)$ is true for $n=k+1$.

$\therefore P(n)$ is true for all $n \in N$ //.

(61)

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17th NOV

7. P.T $a + ar + ar^2 + \dots + ar^{n-1} = \frac{a(r^n - 1)}{r - 1}$, for n is a +ve Integer by using M.I.

Sol:- If $n = 1$.

$$ar^{n-1} = \frac{a(r^n - 1)}{r - 1}$$

$$ar^{1-1} = \frac{a(r^1 - 1)}{r - 1}$$

(62)

$a(1) = a$.

$\therefore P(n)$ is true for $n = 1$.

Let us suppose that $P(n)$ is true for $n = k$.

$$a + ar + ar^2 + \dots + ar^{k-1} = a\frac{(r^k - 1)}{r - 1} \rightarrow (1)$$

$n = k + 1$.

n^{th} term $= ar^{n-1}$

$(k+1)^{th}$ term $= ar^{k+1-1}$

$= ar^k$.

Adding $(k+1)^{th}$ term on B.S. in (1)

$$a + ar + ar^2 + \dots + ar^{k-1} + ar^k = \frac{a(r^k - 1)}{r - 1} + ar^k$$

$$= \frac{ar^k - a}{r - 1} + ar^k$$

$$= \frac{ar^k - a + (r-1)ar^k}{r - 1}$$

$$= \frac{ar^k - a + ar^{k+1} - ar^k}{r - 1}$$

$$= \frac{a(r^{k+1} - 1)}{r - 1}$$

8. P.T $a + ar + ar^2 + \dots + ar^n = \frac{a(r^{n+1} - 1)}{r - 1}$ when n is a non-ve Integer.

Sol:- $n = 0$

$$ar^n = \frac{a(r^n + 1) - 1}{r - 1}$$

$$ar^0 = \frac{a(r^0 + 1) - 1}{r - 1}$$

$a = a\frac{(r - 1)}{r - 1}$

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$a = a$.

$\therefore P(n)$ is true for $n = 0$

Let us suppose that $P(n)$ is true for $n = k$.

$$a + ar + ar^2 + \cdots + ar^k = \frac{a(r^{k+1} - 1)}{r-1} \rightarrow (1)$$

If $n = k+1$,

n^{th} term $= ar^n$

$(k+1)^{th}$ term $= ar^{k+1}$

$(k+1)^{th}$ term adding on B.S in (1)

$$a + ar + ar^2 + \cdots + ar^k + ar^{k+1} = \frac{a(r^{k+1} - 1)}{r-1} + ar^{k+1}$$

$$= \frac{ar^{k+1} - a + ar^{k+2} - ar^{k+1}}{r-1}$$

$$= \frac{ar^{k+2} - a}{r-1}$$

$$= \frac{ar^{(k+1)+1} - a}{r-1}$$

$$= \frac{a(r^{(k+1)+1} - 1)}{r-1}$$

$\therefore P(n)$ is true for all $n = k+1$. /—

9. P.T Every the integer in $1.2.3 + 2.3.4 + \cdots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$

Sol:-

$n = 1$.

$$n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$

$$1(1+1)(1+2) = \frac{1(1+1)(1+2)(1+3)}{4}$$

$$(2)(3) = \frac{2(3)(4)}{4}$$

$$6 = 6$$

$\therefore P(n)$ is true for $n = 1$.

Let us suppose that $P(n)$ is true for $n = k$.

$$1.2.3 + 2.3.4 + \cdots + K(K+1)(K+2) = \frac{k(k+1)(k+2)(k+3)}{\cdot} \rightarrow (1)$$

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If $n = k+1$

n^{th} term $= n(n+1)(n+2)$

$(k+1)^{th}$ term $= (k+1)(k+2)(k+3)$

Adding $(k+1)^{th}$ term on B.S. in (1)

$1.2.3 + 2.3.4 + \dots + k(k+1)(k+2) + (k+1)(k+2)(k+3) = \frac{k(k+1)(k+2)(k+3)}{4} +$ (64) $(k+1)(k+2)(k+3)$

$= (k+1)(k+2)(k+3)\left[\frac{k}{4} + 1\right]$

$= \frac{(k+1)(k+2)(k+3)(k+4)}{4}$

$= \frac{(k+1)(k+1+1)(k+1+2)(k+1+3)}{4}$ //-

10. P.T $3 + 3.5 + 3.5^2 + 3.5^3 + \dots + 3.5^n = \frac{3(5^{n+1}-1)}{4}$ when

n is a non-negative Integer.

Sol:-

If $n = 0$

$3.5^n = \frac{3(5^{n+1}-1)}{4}$

$3.5^0 = \frac{3(5^{0+1}-1)}{4} \Rightarrow 3(1) = \frac{3(4)}{4} \Rightarrow 3 = 3.$

$\therefore P(n)$ is true for $n = 0$

$\therefore$ Let us Suppose that $P(n)$ is true for $n = k$.

$3 + 3.5 + 3.5^2 + 3.5^3 + \dots + 3.5^k = \frac{3(5^{k+1}-1)}{4} \rightarrow (1)$

If $n = (k+1)$

n^{th} term $= 3.5^n$.

$(k+1)^{th} = 3.5^{(k+1)}$

$3 + 3.5 + 3.5^2 + 3.5^3 + \dots + 3.5^k + 3.5^{(k+1)} =$

$= \frac{3(5^{k+1}-1)}{4} + 3.5^{k+1}$

$= \frac{3.5^{k+1} - 3 + 3.5^{k+1}}{4}$

$= \frac{3(5^{k+1} - 1 + 5^{k+1})}{4}$

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$= \frac{3(5^{k+1} \cdot 5^1 - 1)}{4}$

$= \frac{3(5^{k+1} \cdot 5^{1a} - 1)}{4}$

$= \frac{3(5^{(k+1)+1} - 1)}{4}$

$\therefore$ P(n) is true for $n = k+1$

$\therefore$ P(n) is true for all $n \in N$.

(ii) P.T $2 - 2.7 + 2.7^2 - 2.7^3 + \dots + 2.(-7^n) = \frac{1-(-7)^{n+1}}{4}$ when

Even 'n' is a non-ve Integer.

Sol:- If $n = 0$

$2(-7)^n = \frac{1-(-7)^{n+1}}{4}$

$2(-7)^0 = \frac{1-(-7)^{0+1}}{4}$

$2 = \frac{1+7}{4} \Rightarrow 2 = \frac{8}{4}^2 \Rightarrow 2 = 2$

$\therefore$ P(n) is true for $n = 0$.

$\therefore$ Let us assume that P(n) is true for $n = k$.

$2 - 2.7 + 2.7^2 - 2.7^3 + \dots - 2(-7)^k = \frac{1-(-7)^{k+1}}{4} \rightarrow (1)$

If $n = k+1$

n^{th} term $= 2(-7)^{k+1}$

$(k+1)^{th}$ term $= 2(-7)^{k+2}$.

$(k+1)^{th}$ term adding on b.s in Eqn (1)

$2 - 2.7 + 2.7^2 - 2.7^3 + \dots + 2(-7)^k + 2(-7)^{k+2} = \frac{1-(-7)^{k+1}}{4} + 2(-7)^{k+2}$

$= \frac{1-(-7)^{k+1} + 2(-7)^{k+2}}{4}$

$= \frac{1-(-7)^{k+1} + 8(-7)^{k+2}}{4}$

$\Rightarrow \left[\frac{1+[-7)^{k+1}(-1+8)}{4}\right] \Rightarrow \frac{1+[-(-7)^{k+1}(1-8)]}{4}$

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$= \frac{1-(-7)^{k+1}(-7)^1}{4}$

$\therefore$ P(n) is true for $n=1$

$\therefore$ P(n) is true for all $n \in N$ /—

**12. Use M.I P.T n^3-n is divisible by 3. when n is a +ve integers.

Sol:— If $n=1$

$1^3-1 = 1-1=0$

$= 3(0)$

$\therefore$ P(n) is true for $n=1$.

Let us suppose that P(n) is true for $n=k$.

$k^3-k = 3p$ [p is constant] → ①

If $n=k+1$

$(k+1)^3-(k+1) = (k+1)[(k+1)^2-1]$

$= (k+1)[k^2+2k+1-1]$

$= (k+1)[k^2+2k]$

$= k^3+2k^2+k^2+2k.$ [$\because$ Eqn ① ⇒ $k^3-k=3p$, $k^3=3p+k$]

$= k^3+3k^2+2k$

$= 3p+k+3k^2+2k$

$= 3k^2+3k+3p$

$= 3(k^2+k+p)$

$\therefore$ P(n) is true for $n=k+1$ /—

13. P.T n^5-n divisible by 5. when Even n is a non-ve integer.

Sol:— $n=0$

$0^5-0 = 0$

$= 5(0)$

$\therefore$ P(n) is true for $n=0$

Let us suppose that P(n) is true for $n=k$.

$k^5-k = 5p$ → ①

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If $n = k+1$

$(k+1)^5 - (k+1) = (k+1)\left[(k+1)^4 - 1\right]$

$= (k+1)\left[(k+1)(k+1)^3 - 1\right]$

$= (k+1)\left[(k+1)(k^3+3k+3k^2+1) - 1\right]$

$= (k+1)\left[k^4 + 3k^2 + 3k^3 + k + k^3 + 3k + 3k^2 + 1 - 1\right]$

$= (k+1)\left[k^4 + 4k^3 + 6k^2 + 4k\right]$

$= k^5 + 4k^4 + 6k^3 + 4k^2 + k^4 + 4k^3 + 6k^2 + 4k.$

$= k^5 + 5k^4 + 10k^3 + 10k^2 + 4k.$ (67)

$= 5p + k + 5k^4 + 10k^3 + 10k^2 + 4k.$

$= 5k^4 + 10k^3 + 10k^2 + 5k + 5p$

$= 5(k^4 + 2k^3 + 2k^2 + k + p)$ ∎

$\therefore P(n)$ is true for $n = k+1$

14. P.T $4^{n+1} + 5^{2n-1}$ is divisible by 21. By M.I. when n is a true integer.

Sol:- $n = 1.$

$4^2 + 5^{2-1} = 16 + 5$

$= 21$

$= 21(1)$

$\therefore P(n)$ is true for $n = 1.$

Let us suppose that $P(n)$ is true for $n = k.$

$4^{k+1} + 5^{2k-1} = 21p \rightarrow (1)$

If $n = k+1$

$4^{k+2} + 5^{(2k+2)-1} = 4^{k+2} + 5^{2(k+1)-1}$

$= 4^{k+2} + 5^{2k+2-1}$

$= 4^{k+2} + 5^{(2k-1)+2}$

$= 4^{k+2} + 5^{2k-1} \cdot 5^2.$

$= 4^{k+2} + 25\left[21p - 4^{k+1}\right]$

$= 4^{k+2} + 25 \times 21p - 25 \times 4^{k+1}$

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$= 4^{k+1} \cdot 4^1 + 25 \times 21p - 25 \times 4^{k+1}$

$= -21 \times 4^{k+1} + 25 \times 21p$

$= 21[25p - 4^{k+1}]$

$\therefore P(n)$ is true for $n = k+1$ ∎

15) P.T. n is a +ve Integer then 133 divided by $11^{n+1} + 12^{2n-1}$

Sol:- $n = 1$ (68)

$11^{n+1} + 12^{2n-1} = 11^{1+1} + 12^{2(1)-1}$

$= 11^2 + 12^1$

$= 133 (1)$

$\therefore P(n)$ is true for $n = 1$.

Let us suppose that $P(n)$ is true for $n = k$.

$11^{k+1} + 12^{2k-1} = 133p \rightarrow (1)$

If $n = k+1$.

$11^{k+2} + 12^{(2k+2-1)} = 11^{k+2} + 12^{2(k+1)-1}$

$= 11^{k+2} + 12^{2k+2-1}$

$= 11^{k+2} + 12^{(2k-1)+2}$

$= 11^{k+2} + 12^{(2k+1} \cdot 12^2$

$= 11^{k+2} + 144[133p - 11^{k+1}]$

$= 11^{k+2} + 144 \times 133p - 144 \times 11^{k+1}$

$= 11^{k+1} \cdot 11^1 + 144 \times 133p - 144 \times 11^{k+1}$

$= -133 \times 11^{k+1} + 144 \times 133p$

$= 133[144p - 11^{k+1}]$

$\therefore P(n)$ is true for $n = k+1$. ∎

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16. use M.I To P.T $2^n < n!$ for Every +ve Integer. n. with $n \geq 4$.

Sol:- $P(n) = 2^n < n!$

If $n=4$.

$2^4 < 4!$

$16 < 24$.

$\therefore P(n)$ is true for $n=4$.

Let us suppose that $P(n)$ is true for $n=k$.

$\therefore 2^k < k!$ → ①

If $n = k+1$.

$2^{k+1} < k+1! = 2^{k+1} = 2^k \cdot 2$

$< k! \cdot 2$

$< k(k-1) \cdots 1 \cdot 2$

$< (k+1)!$

$\therefore P(n)$ is true for $n=k+1$ //—

17. The Harmonic number. H_j, $j = (1, 2, 3 \cdots)$ are defined by $H_j = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{j}$ and.

$H_4 = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{25}{12}$. use M.I To S.T $H_{2^n} \geq 1 + \frac{n}{2}$ when n is a non-ve Integer.

Sol:- Given $H_{2^n} \geq 1 + \frac{n}{2}$

If $n=0$

$H_{2^0} \geq 1 + \frac{0}{2}$.

$H_1 \geq 1$

Since, $H_j = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{j}$

$H_1 = \frac{1}{1} = 1$.

$\therefore P(n)$ is true for $n=0$.

Let us suppose that $P(n)$ is true for $n=k$.

$H_{2^k} \geq 1 + \frac{k}{2}$ → ①

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If $n = k+1$

Let $H_{2^{k+1}} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{2^k} + \cdots + \frac{1}{2^{k+1}}$

$= H_{2^k} + \cdots + \frac{1}{2^{k+1}}$ $\left[\because H_{2^k} \geq 1 + \frac{k}{2}\right]$

$\geq 1 + \frac{k}{2} + \cdots + \frac{1}{2^{k+1}}$

$\geq 1 + \frac{k}{2} + \cdots + \frac{1}{2^k \cdot 2}$ [∵ we taking only constant value]

$\geq 1 + \frac{k}{2} + \frac{1}{2}$

$\geq \frac{k+1}{2} + 1$

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$\therefore P\ H_{2^{k+1}} \geq 1 + \frac{(k+1)}{2}$

$P(n)$ is true for $n = k+1$

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UNIT - III

19th Nov

Binomial Expansion.

Binomial theorem :—

If x, y are two Binomial terms and 'n' is the non-negative Integer then. (71)

$$(x+y)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1}y + {}^nC_2 x^{n-2}y^2 + \cdots \pm {}^nC_r x^{n-r}y^r + \cdots + {}^nC_n y^n.$$

(or)

$$= \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \cdots \pm \binom{n}{r}x^{n-r}y^r + \cdots + {}^nC_n y^n$$

Note :— 1. In a Binomial theorem $C_0, C_1 \cdots C_n$ are Co-efficients of x & y

2. ${}^nC_r = \binom{n}{r} = \frac{n!}{(n-r)!\,r!}$

3. $n! = n(n-1)(n-2)\cdots 1.$

N^{th} term & $(r+1)^{th}$ term in Binomial Expansion :—

n^{th} term & $(r+1)^{th}$ term $= T_{r+1} = {}^nC_r x^{n-r} y^r.$

Ex :— $T_6 = {}^nC_5 x^{n-5} y^5.$

$T_{25} = {}^nC_{24} x^{n-24} y^{24}.$

Theorem :—

State and Prove Binomial Expansion :—

Statement :— If x, y are two terms then

$$(x+y)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1}y + {}^nC_2 x^{n-2}y^2 + \cdots + {}^nC_r x^{n-r}y^r + \cdots + {}^nC_n y^n$$

Proof :— This theorem is Proved by mathematical Induction.

$$P(n) = (x+y)^n = {}^nC_0 x^n + {}^nC_1 x^{n-1}y + \cdots + {}^nC_n y^n.$$

If $n = 1$

$$(x+y)^1 = {}^1C_0 x^1 + {}^1C_1 x^0 y + \cdots + {}^1C_1 y^1.$$

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$(x+y)^1 = x+y$.

$\therefore P(n)$ is true for $n=1$.

Let us suppose that $P(n)$ is true for $n=k$.

$\therefore (x+y)^k = {}^kC_0 x^k + {}^kC_1 x^{k-1}y + {}^kC_2 x^{k-2}y^2 + \cdots + {}^kC_k y^k \rightarrow (1)$

If $n = k+1$.

$(x+y)^{k+1} = (x+y)^k (x+y)$ (72)

$= [{}^kC_0 x^k + {}^kC_1 x^{k-1}y + {}^kC_2 x^{k-2}y^2 + \cdots + {}^kC_k y^k](x+y)$

$= [{}^kC_0 x^{k+1} + {}^kC_1 x^k y + {}^kC_2 x^{k-1}y^2 + \cdots + {}^kC_k xy^k] +$

$[{}^kC_0 x^k y + {}^kC_1 x^{k-1}y^2 + {}^kC_2 x^{k-2}y^3 + \cdots + {}^kC_k y^{k+1}]$

$= [{}^kC_0 x^{k+1} + ({}^kC_1 + {}^kC_0)x^k y + ({}^kC_2 + {}^kC_1)x^{k-1}y^2 + \cdots + {}^kC_k y^{k+1}]$

$[\because {}^nC_0 = {}^{n+1}C_0$
$\quad {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r]$

$= (k+1)C_0 x^{k+1} + (k+1)C_1 x^k y + \cdots + (k+1)C_{k+1} y^{k+1}$

$\therefore P(n)$ is true for $n = k+1$,

$\therefore P(n)$ is true for all $n \in N$ /-

Theorem :-

★ State and Prove Pascal Identity :- (81)

If n is a non-negative Integer and r is the selection of numbers then ${}^nC_r = {}^{n-1}C_r + {}^{n-1}C_{r-1}$

$\binom{n}{r} = \binom{n-1}{r} + \binom{n-1}{r-1}$ (81)

Proof :- Given that n is two Integer.

r is selection number.

Now we have to s.t ${}^nC_r = {}^{n-1}C_r + {}^{n-1}C_{r-1}$.

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R.H.S $^{n-1}C_r + {}^{n-1}C_{r-1}$.

$$\frac{(n-1)!}{(n-1-r)!\,r!} + \frac{(n-1)!}{[(n-1)-(r-1)]!\,(r-1)!}$$

$$(n-1)!\left[\frac{1}{(n-r-1)!\,r!} + \frac{1}{[n-r]!\,(r-1)!}\right]$$

$$(n-1)!\left[\frac{1}{(n-r-1)!\,r!} + \frac{1}{(n-r)!\,(r-1)!}\right]$$ (73)

$$(n-1)!\left[\frac{1}{(n-r-1)(n-r-2)\cdots 1\; r(r-1)(r-2)\cdots 1} + \right.$$

$$\left. \frac{1}{(n-r)(n-r-1)\cdots 1\;(r-1)(r-2)\cdots 1}\right]$$

$$(n-1)!\left[\frac{1}{(n-r-1)!\,r(r-1)!} + \frac{1}{(n-r)(n-r-1)!\,(r-1)!}\right]$$

$$\frac{(n-1)!}{(n-r-1)!\,(r-1)!}\left[\frac{1}{r} + \frac{1}{n-r}\right]$$

$$\frac{(n-1)!}{(n-r-1)!\,(r-1)!}\left[\frac{(n-r+r)}{r(n-r)}\right]$$

$$\frac{(n-1)!}{(n-r-1)!\,(r-1)!}\left[\frac{n}{r(n-r)}\right]$$

$$\frac{n(n-1)!}{(n-r)(n-r-1)!\,(r-1)!}$$

$$= \frac{n!}{(n-r)!\,r!}$$

$$\Rightarrow {}^nC_r = \text{LHS}$$ ✓

Problems :—

1) what is the Expansion of $(x+y)^4$.

Sol:- $(x+y)^4 = {}^4C_0 x^4 + {}^4C_1 x^3y + {}^4C_2 x^2y^2 + {}^4C_3 xy^3 + {}^4C_4 y^4$.

$${}^4C_0 = {}^4C_4 = 1$$

$${}^4C_1 = \frac{4!}{3!\,1!} = \frac{4\times 3\times 2\times 1}{3\times 2\times 1} = 4.$$

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$${}^4C_2 = \frac{4!}{2!2!} = \frac{4\times3\times2\times1}{2\times1\times2\times1} = 6$$

$${}^4C_3 = \frac{4!}{1!3!} = \frac{4\times3\times2\times1}{3\times2\times1} = 4$$

$\therefore (x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$ //-

2) what is the co-efficient of $x^{12}.y^{13}$ in the Expansion of $(2x-3y)^{25}$

Sol:- Given Expansion $(2x-3y)^{25}$.

$x = 2x$, $y = -3y$.

$n = 25$

$T_{r+1} = {}^nC_r\, x^{n-r} y^r.$

$= {}^{25}C_r\, x^{25-r} y^r.$

$= {}^{25}C_{13}\, (2x)^{25-13} \cdot (-3y)^{13}$

$= {}^{25}C_{13}\, (2)^{12} \cdot x^{12} (-3)^{13} y^{13}$

$= {}^{25}C_{13}\, (2)^{12} (-3)^{13} \cdot x^{12} y^{13}.$

The co-eff of $x^{12}.y^{13}$ is ${}^{25}C_{13}\,(2)^{12}(-3)^{13}$ //-

3) find the co-efficient of $x^{101}.y^{99}$ in the Expansion $(2x-3y)^{200}$

Sol:- Given Expansion $(2x-3y)^{200}$

$x = 2x$, $y = -3y$

$n = 200$

$T_{r+1} = {}^nC_r\, x^{n-r} y^r.$

$= {}^{200}C_r\, x^{200-r} y^r.$

$= {}^{200}C_{99}\, (2x)^{200-99} \cdot (-3y)^{99}.$

$= {}^{200}C_{99}\, (2)^{101} \cdot x^{101} (-3)^{99} y^{99}.$

$= {}^{200}C_{99}\, (2)^{101} (-3)^{99} \cdot x^{101} y^{99}.$

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$\therefore$ The co-eff of $x^{101}.y^{99}$ in $(2x-3y)^{200}$ is $^{200}C_{99}(2)^{101}(-3)^{99}$ /-

4) find the co-eff of $x^5.y^8$ in $(x+y)^{13}$

Sol:- Given expansion $(x+y)^{13}$

$x = x, \quad y = 13.$

$n = 13.$

$T_{r+1} = {}^nC_r\, x^{n-r} y^r.$ (75)

$= {}^{13}C_r\, x^{13-r} y^r.$

$= {}^{13}C_8\, x^{13-8} y^8.$

$= {}^{13}C_8\, x^5 y^8.$

$\therefore$ The co-eff of $x^5.y^8$ is $^{13}C_8$ /-

5) what is the co-eff of x^7 in $(1+x)^{11}$.

Sol:- Given Expansion $(1+x)^{11}$

$x = 1, \quad y = x$

$n = 11$

$T_{r+1} = {}^nC_r\, x^{n-r} y^r.$

$= {}^{11}C_r \cdot 1^{11-7} (x)^7$

$= {}^{11}C_7 \cdot 1^{11-7} x^7.$

$= {}^{11}C_7 \cdot x^7.$

$\therefore$ The co-eff of x^7 is $^{11}C_7 (x)^7$ /-

6. Give a formula for the co-eff. of x^k in the Expansion of $\left(x^2 + \frac{1}{x}\right)^{100}$ where 'k' is a integer.

Sol:-

$x = x^2, \quad y = 1/x, \quad n = 100.$

$T_{r+1} = {}^nC_r\, x^{n-r} y^r.$

$= {}^{100}C_r\, x^{100-r} y^r.$

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$= {}^{100}C_r (1)^{100-r} (1/x)^r$.

$= {}^{100}C_r (1)^{100-r} (x)^{-r}$

Put $-r = k$.

$\therefore {}^{100}C_{-k} (1)^{100+k} (x)^k$ /-

7) Give a formula for the co-eff of x^k in the Expansion of $(x + \frac{1}{x})^{100}$

(76)

Sol:- $x = x$, $y = 1/x$, $n = 100$

$T_{r+1} = {}^nC_r x^{n-r} y^r$.

$= {}^{100}C_r (x)^{100-r} (1/x)^r$.

$= {}^{100}C_r x^{100-r} (x)^{-r}$.

Put $-r = k$.

$= {}^{100}C_{-k} x^{100+k} (x)^k$

$= {}^{100}C_{-k} x^{100+2k}$ /—

Sequence :— A function f from Z^+ to R is said to be Sequence (or)

$f : Z^+ \to R$.

In a function Domain is the Integer then it is called Sequence, and it is denoted by $\{S_n\}$ or $\langle S_n \rangle$ (or) (S_n).

Bounded Sequence :—

Bounded above Sequence :— A Sequence is said to be bounded above Sequence if there Exist $k_1 \in Z^+$ such that $|S_n| < k_1$

Bounded below Sequence :— A Sequence $\{S_n\}$ is said to be bounded below Sequence if there Exist $k_2 \in Z^+$ such that $|S_n| > k_2$

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Bounded Sequence :— A Sequence $\{s_n\}$ is said to be bounded Sequence if their Exist $k \in z^+$ such that $|s_n| \leq k$.

→ If $\{s_n\}$ is bounded Sequence then $|s_n| \leq k$

$\Rightarrow -k < s_n < k$.

$\therefore s_n \in [-k, k)$

(77)

Limit of a Sequence :— A Sequence $\{s_n\}$ is said to be limit Sequence. If their Exist $\epsilon > 0$ such that $|s_n - l| < \epsilon$ for all $n \in z$. It is denoted by $\lim_{n \to \infty} s_n = l$. (81)

$\lim_{n \to \infty} s_n = l$.

Types of Sequences :—

1. Convergent, 2. Divergent 3. oscillating Sequences.

20th NOV

Convergent Sequence :— A Sequ. is said to be convergent Sequence if their Exist only one limit point.

(81)

If let l of $\{s_n\}$ is a convergent Sequence their Exist $\epsilon > 0$. atleast $m \in z^+$ $\exists$ $|s_n - l) < \epsilon$, $n \geq m$.

→ If $\{s_n\}$ is convergent then $|s_n - l| < \epsilon$

i.e: $\lim s_n = l$.

Divergent Sequence :— A Seq^n $\{s_n\}$ is said to be divergent. If their Exist more than one limit points.

Oscillating Sequence :— A Seq^n is said to be oscillating If their Exist two limit points.

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Recurrance Relation :—

Problems :—

1) Let the $\{a_n\}$ be a sequence that satisfies the relation $a_n = a_{n-1} - a_{n-2}$ for $n = 2, 3, 4$ and $a_0 = 3, a_1 = 5$. Then find a_2 and a_3.

Sol:— Given relations $a_n = a_{n-1} - a_{n-2}$

$n = 2, 3, 4$ & $a_0 = 3, a_1 = 5$.

$a_n = a_{n-1} - a_{n-2}$. (78)

$n = 2, \; a_2 = a_1 - a_0$

$a_2 = 5 - 3 = 2$.

$n = 3; \; a_3 = a_2 - a_1$

$a_3 = 2 - 5 = -3$

$n = 4; \; a_4 = a_3 - a_2$

$a_4 = -3 - 2 = -5$ //—

2) find the first 5 terms of the sequence defined by each of recurrance relation & initial conditions $a_n = 6a_{n-1}, \; a_0 = 2$

Sol:— Given $a_n = 6a_{n-1}, \; a_0 = 2$

$n = 1, \; a_1 = 6a_0 = 6(2) = 12$

$n = 2, \; a_2 = 6a_1 = 6(12) = 72$

$n = 3, \; a_3 = 6a_2 = 6(72) = 432$.

$n = 4, \; a_4 = 6a_3 = 6(432) = 2592$

$n = 5, \; a_5 = 6a_4 = 6(2592) = 15552$. //—

3) find the 1st 10 terms of the sequence. defined by $a_n = a_{n-1} + 3a_{n-2}, \; a_0 = 1, \; a_1 = 2$

Sol:— Given. $a_n = a_{n-1} + 3a_{n-2}, \; a_0 = 1, \; a_1 = 2$.

$n = 2; \; a_2 = a_{n-1} + 3a_{n-2}$

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$= a_1 + 3a_0 \Rightarrow 2 + 3(1) \Rightarrow 5.$

$n=3 \Rightarrow a_3 = a_{3-1} + 3a_{3-2} \Rightarrow a_2 + 3a_1 \Rightarrow 5+6 = 11$

$n=4 \Rightarrow a_4 = a_{4-1} + 3a_{4-2}$

$= a_3 + 3a_2 = 11 + 3(5)$

$= 26$

$n=5 \Rightarrow a_5 = a_{5-1} + 3a_{5-2} \Rightarrow a_4 + 3a_3 = 26 + 3(11)$

$= 59.$

$n=6 \Rightarrow a_6 = a_{6-1} + 3a_{6-2} \Rightarrow a_5 + 3a_4 = 59 + 3(26) = 137$

$n=7 \Rightarrow a_7 = a_{7-1} + 3a_{7-2} \Rightarrow a_6 + 3a_5 = 137 + 3(59) = 314.$

$n=8 \Rightarrow a_8 = a_{8-1} + 3a_{8-2} \Rightarrow a_7 + 3a_6 = 314 + 3(137) = 725$

$n=9 \Rightarrow a_9 = a_{9-1} + 3a_{9-2} \Rightarrow a_8 + 3a_7 = 725 + 3(314) = 1667$

$n=10 \Rightarrow a_{10} = a_{10-1} + 3a_{10-2} \Rightarrow a_9 + 3a_8 = 1667 + 3(725) = 3842/-$

4) $a_n = na_{n-1} + n^2 a_{n-2}$, $a_0 = 1$, $a_1 = 1$. find 1st 5 terms.

Sol:- $n=2, \quad a_2 = 2a_1 + (4)a_0$

$= 2(1) + 4(1)$

$= 6$

$n=3, \quad a_3 = 3a_2 + (9)a_1 \Rightarrow 3(6) + 9(1)$

$\Rightarrow 18+9 = 27.$

$n=4, \quad a_4 = 4a_3 + 16a_1 \Rightarrow 4(27) + 16(6) = 108 + 16 = 204$

$n=5, \quad a_5 = 5a_4 + 25a_3 \Rightarrow 5(204) + 25(27) = 1020 + 675 = 1695$

$n=6, \quad a_6 = 6a_5 + 36a_4 \Rightarrow 6(1695) + 36(204) = 10170 + 7344 = 17514/-$

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5) Let $a_n = 2^n + 5.3^n$, $n = 0, 1, 2 \ldots$ then

1. find a_0, a_1, a_2, a_3 & a_4.
2. S.T $a_2 = 5.a_1 - 6a_0$, $a_3 = 5a_2 - 6a_1$, $a_4 = 5a_3 - 6a_2$
3. S.T $5a_{n-1} - 6a_{n-2}$ is a true value. for all integers. with $n \geq 2$

(80)

Sol:- I. Given that $a_n = 2^n + 5.3^n$, $n = 0, 1, 2 \ldots$

$n = 0$, $a_0 = 2^0 + 5.3^0$

$a_0 = 1 + 5.1 = 6$

$n = 1 \Rightarrow a_1 = 2^1 + 5.3^1$;

$= 2 + 5.3$

$a_1 = 2 + 15 = 17.$

$n = 2 \Rightarrow a_2 = 2^2 + 5.3^2$

$= 4 + 5(9)$

$a_2 = 4 + 45 = 49.$

$n = 3 \Rightarrow a_3 = 2^3 + 5.3^3$;

$= 8 + 5(27)$

$= 8 + 135$

$a_3 = 143$

$n = 4 \Rightarrow a_4 = 2^4 + 5.3^4$

$= 16 + 5(81)$

$= 16 + 405$

$a_4 = 421$

II. S.T $a_2 = 5a_1 - 6a_0$; $a_3 = 5a_2 - 6a_1$

$49 = 5(17) - 6(6)$; $143 = 5(49) - 6(17)$

$49 = 85 - 36$; $143 = 245 - 102$

$49 = 49$; $143 = 143$

$a_4 = 5a_3 - 6a_2$

$421 = 5(143) - 6(49)$

$421 = 715 - 294$

$421 = 421.$

III. S.T $5a_{n-1} - 6a_{n-2}$ is a true Integer with $n \geq 2$.

Let $a_n = 5a_{n-1} - 6a_{n-2}$.

If $n = 2 \Rightarrow a_2 = 5a_{2-1} - 6a_{2-2}$.

$a_2 = 5a_1 - 6a_0 = 5(17) + 6(6)$

$49 = 85 + 36$

$49 = 49$ I

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If $n=3$; $a_3 = 5a_{3-1} - 6a_{3-2}$

$= 5a_2 - 6a_1 \Rightarrow 5(49) - 6(17)$

$143 \Rightarrow 245 - 102$

$143 = 143.$

If $n=4$; $a_4 = 5a_{4-1} - 6a_{4-2}$

$= 5a_3 - 6a_2 \Rightarrow 5(143) - 6(49)$

$421 = 715 - 294$

$421 = 421.$

$\therefore 5a_{n-1} - 6a_{n-2}$ values are +ve integers /—

6. ~~Find~~ S.T. the sequence $\{S_n\}$ is a solution of the relation $a_n = 2 \cdot a_{n-1} - a_{n-2}$ for $n = 2, 3, 4 \ldots$ If $a_n = 2^n$ when n is a non-ve integer.

Sol:- Given relation $a_n = 2a_{n-1} - a_{n-2}$ $n = 2, 3, 4 \ldots$

$n=2$, $a_2 = 2a_1 - a_0$

$= 2(2) - 1 = 3$

$n=3$, $a_3 = 2a_2 - a_1$

$= 2(3) - 2 = 4$

$n=4$; $a_4 = 2a_3 - a_2$

$= 2(4) - 3 = 5$

$n=5$; $a_5 = 2a_4 - a_3$

$= 2(5) - 4$

$= 10 - 4 = 6$ /—

$\therefore a_n = 2^n$

$a_0 = 2^0 = 1$

$a_1 = 2^1 = 2.$

** Solving linear recurrence relation :—

A linear homogenious relation of degree 'k' with constant co-efficient in the form of $a_n = c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k}$.

where $c_1, c_2 \ldots c_k$ are real numbers. and $c_k \neq 0$

Initial conditions are $a_0 = c_0$, $a_1 = c_1$, $a_2 = c_2 \ldots a_{k-1} = c_{k-1}$

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Note:- 1. Let c_1, c_2 are real numbers suppose that $r^2 - c_1 r - c_2 = 0$ two roots are r_1, r_2 then the sequence $\{a_n\}$ is a solution of the relation $a_n = c_1 \cdot a_{n-1} + c_2 \cdot a_{n-2}$ iff $a_n = \alpha_1 (r_1)^n + \alpha_2 (r_2)^n$ for $n = 0, 1, 2 \ldots$ and α_1, α_2 are constants.

Problems:- (82)

1) what is the solution of relation $a_n = a_{n-1} + 2a_{n-2}$ when $a_0 = 2$, $a_1 = 7$.

Sol:- Given $a_n = a_{n-1} + 2a_{n-2}$, $a_0 = 2$, $a_1 = 7$.

$r^2 = r + 2$

$r^2 - r - 2 = 0$

$r^2 - 2r + r - 2 = 0$

$r(r-2) + 1(r-2) = 0$

$(r-2)(r+1) = 0$

$r - 2 = 0, \quad r + 1 = 0$

$r = 2, \quad r = -1$

$r = -1, 2$.

$\therefore$ roots are $2, -1$.

$\therefore r_1 = 2, r_2 = -1$.

Since, the sequence $\{a_n\}$ is a solution to the relation iff $a_n = \alpha_1 (r_1)^n + \alpha_2 (r_2)^n$.

$a_n = \alpha_1 (2)^n + \alpha_2 (-1)^n$.

If $n = 0$, $a_0 = \alpha_1 (2)^0 + \alpha_2 (-1)^0$ [$\because 2^0 = 1$, $-1^0 = 1$]

$a_0 = \alpha_1 + \alpha_2$

$\therefore \alpha_1 + \alpha_2 = 2 \rightarrow (1)$

If $n = 1$, $a_1 = \alpha_1 (2)^1 + \alpha_2 (-1)^1$

$a_1 = 2\alpha_1 - \alpha_2$

$2\alpha_1 - \alpha_2 = 7 \rightarrow (2)$

$r^2 = r + 2$

$r^2 - r - 2 = 0$

$r^2 - 2r + r - 2 = 0$

$r(r-2) + 1(r-1) = 0$

$r_1 = 2, \quad r = -1$

roots are $2, -1$.

$\therefore r_1 = 2, r_2 = -1$

$a_n = \alpha_1 (r_1)^n + \alpha_2 (r_2)^n$.

$\Rightarrow \alpha_1 (2)^n + \alpha_2 (-1)^n$

$\Rightarrow$ If $n = 0 \Rightarrow a_0 = \alpha_1 (2)^0 + \alpha_2 (-1)^0$

$= \alpha_1 + \alpha_2$

$2 = \alpha_1 + \alpha_2 \rightarrow (1)$

If $n = 1$, $a_1 = \alpha_1 (2)^1 + \alpha_2 (-1)^1$

$a_1 = 2\alpha_1 - \alpha_2$

$2\alpha_1 - \alpha_2 = a_1 \rightarrow (2)$

Now solving (1) & (2)

$\alpha_1 + \alpha_2 = 2$

$2\alpha_1 - \alpha_2 = 7$

$3\alpha_1 = 9$

$\alpha_1 = 9/3$

$\alpha_1 = 3$.

$3 + \alpha_2 = 2$

$\alpha_2 = 2 - 3$

$\alpha_2 = -1$

$a_n = \alpha_1 (r_1)^n + \alpha_2 (r_2)^n$

$a_n = 3(2)^n + (-1)(1)^n$

$= 2^n + 1^n$

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Now solving ① & ②

$\alpha_1 + \alpha_2 = 2$

$2\alpha_1 - \alpha_2 = 7$

$3\alpha_1 = 9$

$\alpha_1 = 9/3 = 3.$

from ① $\Rightarrow \alpha_1 + \alpha_2 = 2$

$3 + \alpha_2 = 2$

$\alpha_2 = 2 - 3$

$\alpha_2 = (-1)$

∴ solution of the given Eqn is $a_1 = \alpha_1(\gamma_1)^n + \alpha_2(\gamma_2)^n$

$a_n = 3(2)^n + -1(-1)^n.$

$a_n = 3(2)^n + (1)^n$ //-

2) what is the solution of $a_n = 6a_{n-1} + 9a_{n-2}$ with initial conditions $a_0 = 1$ & $a_1 = 6$.

Sol:- Given $a_n = 6a_{n-1} + 9a_{n-2}$, $a_0 = 1$ & $a_1 = 6$

$\gamma^2 = 6\gamma + 9$

$\gamma^2 - 6\gamma - 9 = 0$

~~$\gamma^2 - 3\gamma - 3\gamma - 9 = 0$~~

~~$\gamma(\gamma - 3) - 3$~~

$\gamma = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1,\ b = -3,\ c = -9.$

$\gamma = \frac{3 \pm \sqrt{9 + 36}}{2} \Rightarrow \frac{3 \pm \sqrt{45}}{2} \Rightarrow \frac{3 \pm 3\sqrt{5}}{2} \Rightarrow \frac{3(1 \pm \sqrt{5})}{2}$

$\gamma_1 = \frac{3(1+\sqrt{5})}{2}$, $\gamma_2 = \frac{3(1-\sqrt{5})}{2}$

Since the sequence $\{s_n\}$ is a solution of the relation.

$a_n = \alpha_1(\gamma_1)^n + \alpha_2(\gamma_2)^n.$

$a_n = \alpha_1\left(\frac{3(1+\sqrt{5})}{2}\right)^n + \alpha_2\left(\frac{3(1-\sqrt{5})}{2}\right)^n.$

If $n = 0$, $a_0 = \alpha_1\left(3\frac{(1+\sqrt{5})}{2}\right)^0 + \alpha_2\left(3\frac{(1-\sqrt{5})}{2}\right)^0$

$a_0 = \alpha_1 + \alpha_2$

$\alpha_1 + \alpha_2 = 1 \longrightarrow ①$

If $n = 1$, $a_1 = \alpha_1\left(3\frac{(1+\sqrt{5})}{2}\right)^1 + \alpha_2\left(\frac{3(1-\sqrt{5})}{2}\right)^1.$

$= \frac{3}{2}(1+\sqrt{5})\alpha_1 + \frac{3}{2}(1-\sqrt{5})\alpha_2 = 6 \longrightarrow ②$

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from Eqn ① & ②

$\alpha_1 + \alpha_2 = 1,$

$\frac{3}{2}(1+\sqrt{5})\alpha_1 + \frac{3}{2}(1-\sqrt{5})\alpha_2 = 6.$

$\frac{3}{2}[(1+\sqrt{5})\alpha_1 + (1-\sqrt{5})\alpha_2] = 6.$

$(1+\sqrt{5})\alpha_1 + (1-\sqrt{5})\alpha_2 = 4 \rightarrow ②$

from Eqn ① × $(1+\sqrt{5})$

$(1+\sqrt{5})\alpha_1 + (1+\sqrt{5})\alpha_2 = 1$

$(1+\sqrt{5})\alpha_1 + (1-\sqrt{5})\alpha_2 = 4$

$-\quad\quad -\quad\quad -$

$(1+\sqrt{5})\alpha_2 - (1-\sqrt{5})\alpha_2 = -3.$

$\alpha_2(1+\sqrt{5}-1+\sqrt{5})\alpha_2 = -3$

$\alpha_2(2\sqrt{5}) = -3$

$\therefore \alpha_2 = -3/2\sqrt{5}.$

from α_2 value sub in Eqn ①

$\alpha_1 - \frac{3}{2\sqrt{5}} = 1.$

$\alpha_1 = 1 + \frac{3}{2\sqrt{5}}$

$\alpha_1 = 1 + \frac{3}{2\sqrt{5}}$

Given soln is Eqn. is $\alpha_1 r_1^n + \alpha_2 r_2^n$

$a_n = \left(1 + \frac{3}{2\sqrt{5}}\right)\left(3\left(\frac{1+\sqrt{5}}{2}\right)\right)^n + \frac{-3}{2\sqrt{5}}\left(3\frac{(1-\sqrt{5})}{2}\right)^n$

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22nd NOV

3) what is the solution of the recurrance relation $a_n = 6a_{n-1} - 9a_{n-2}$, $a_0 = 2$, $a_1 = 7$.

Sol:- $r^2 = r + 2$

$r^2 = 6r - 9$

$r^2 - 6r + 9 = 0$

$r^2 - 3r - 3r + 9 = 0$

$r(r-3) - 3(r-3) = 0$

$r_1 = 3, r_2 = 3$.

$(r-3)(r-3) = 0$

$r_1 = 3, 3$

(85)

∴ Roots are 3, 3

$\therefore r_1 = 3, r_2 = 3$

Since the sequence $\{a_n\}$ is the solution to the relation

iff $a_n = \alpha_1 (r_1)^n + \alpha_2 (r_2)^n$

$a_n = \alpha_1 (3)^n + \alpha_2 (3)^n$

If $n = 0$, $a_0 = \alpha_1 (3)^0 + \alpha_2 (3)^0$

$a_0 = \alpha_1 + \alpha_2 \Rightarrow \alpha_1 + \alpha_2 = 2 \rightarrow (1)$

If $n = 1$, $a_1 = \alpha_1 (3)^1 + \alpha_2 (3)^1$

$a_1 = 3\alpha_1 + 3\alpha_2$

$3\alpha_1 + 3\alpha_2 = 7$

$3(\alpha_1 + \alpha_2) = 7$

$\alpha_1 + \alpha_2 = 7/3 \rightarrow (2)$

From (1) & (2)

$\alpha_1 + k = 2$

$\alpha_1 = 2 - k$; $(2-k) + \alpha_2 = 7/3$

$a_n = \alpha_1 (r_1)^n + \alpha_2 (r_2)^n$

$a_n = 3(2-k)^n + 9k \cdot 3^n$

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4) find the solution to the recurrence relation with the initial conditions

$a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$

$a_0 = 2$, $a_1 = 5$, & $a_2 = 15$.

Sol:- $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$

$r^3 = 6r^2 - 11r + 6$

$r^3 - 6r^2 - 11r - 6 = 0$

	1	-6	-11	6
0		1	-5	-6
	1	-5	-6	0

$(r+1)(r^2 - 5r + 6) = 0$

$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$; $a = 1$, $b = -5$, $c = 6$

$r = \frac{5 \pm \sqrt{5^2 - 4(1)(6)}}{2} \Rightarrow \frac{5 \pm \sqrt{25 - 24}}{2} = \frac{5 \pm \sqrt{1}}{2}$

$\Rightarrow \frac{5+1}{2}, \frac{5-1}{2}$ $= \frac{5 \pm 1}{2}$

$\Rightarrow \frac{6}{2} 3, \frac{4}{2} 2$ $\Rightarrow 3, 2.$

$(r_1+1)(r_2+3)(r_3+2)$

$\therefore r_1 = -1$, $r_2 = -3$, $r_3 = -2$.

$\therefore a_n = \alpha_1(r_1)^n + \alpha_2(r_2)^n + \alpha_3(r_3)^n$.

If $n = 0$; $a_0 = \alpha_1(-1)^0 + \alpha_2(-3)^0 + \alpha_3(-2)^0$

$a_0 = \alpha_1 + \alpha_2 + \alpha_3$

$\alpha_1 + \alpha_2 + \alpha_3 = 2 \rightarrow (1)$

If $n = 1$; $a_1 = \alpha_1(-1)^1 + \alpha_2(-3)^1 + \alpha_3(-2)^1$

$a_1 = -\alpha_1 - 3\alpha_2 - 2\alpha_3$

~~$-\alpha_1 - 3\alpha_2 - 2\alpha_3 = 5 \rightarrow (2)$~~

~~$-(\alpha_1 + 3\alpha_2 + 2\alpha_3) = 5 \rightarrow (2)$~~ $\alpha_1 + 3\alpha_2 + 2\alpha_3 = -5 \rightarrow (2)$

If $n = 2$; $a_2 = \alpha_1(-1)^2 + \alpha_2(-3)^2 + \alpha_3(-2)^2$.

$15 = \alpha_1 + 9\alpha_2 + 4\alpha_3 \rightarrow (3)$

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Solve ① & ②

$\alpha_1 + \alpha_2 + \alpha_3 = 2$

$\alpha_1 + 3\alpha_2 + 2\alpha_3 = 5$

$-2\alpha_2 - \alpha_3 = -3 \rightarrow ④$

Solving ② & ③

$\alpha_1 + 3\alpha_2 + 2\alpha_3 = 5$

$\alpha_1 + 9\alpha_2 + 4\alpha_3 = 15$

$-6\alpha_2 - 2\alpha_3 = -10 \rightarrow ⑤$

from ④ & ⑤

$4 \times ② \Rightarrow -4\alpha_2 - 2\alpha_3 = -6$

$⑤ \Rightarrow -6\alpha_2 - 2\alpha_3 = -10$

$+2\alpha_2 = 4,$

$\therefore \boxed{\alpha_2 = +2}$

(81)

sub α_2 in ④

$-2(+2) - \alpha_3 = -3$

$-4 - \alpha_3 = -3$

$-\alpha_3 = -3 + 4$

$\therefore \boxed{\alpha_3 = -1}$

subs α_2 & α_3 in ②

$\alpha_1 + 3\alpha_2 + 2\alpha_3 = 5$

$\Rightarrow \alpha_1 + 3(2) + 2(-1) = 5$

$\alpha_1 + 6 - 2 = 5$

$\alpha_1 = 5 - 4$

$\alpha_1 = 1$

$\therefore a_n = (-1)(1)^n + 2 \cdot 3^n + (-1)(2)^n$

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5) find the solution of $a_n = 2a_{n-1} + 5a_{n-2} - 6a_{n-3}$ with $a_0 = 7$, $a_1 = -4$, $a_2 = 8$.

Soln: $a_n = 2a_{n-1} + 5a_{n-2} - 6a_{n-3}$

$r^3 = 2r^2 + 5r - 6$.

$r^3 - 2r^2 - 5r + 6 = 0$

$(r+1)(r^2 - r - 6) = 0$

$r^2 - r - 6 = 0$

$r^2 - 3r + 2r - 6 = 0$

$r(r-3) + 2(r-3) = 0$

$(r-3)(r+2) = 0$

$r_1 = 1 \quad | \quad r_2 = 3 \quad | \quad r_3 = -2$

88

~~$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$~~

~~$a=1, b=-1, c=-6$.~~

~~$\frac{1 \pm \sqrt{(-1)^2 - 4(1)(-6)}}{2(1)}$~~

~~$\frac{1 \pm \sqrt{1+24}}{2} \Rightarrow \frac{1 \pm \sqrt{25}}{2} \Rightarrow \frac{1 \pm 5}{2} \Rightarrow \frac{1+5}{2} \Rightarrow 3$~~

~~$\Rightarrow \frac{1-5}{2} \Rightarrow -2$~~

~~$(\alpha_1 + 1)(\alpha_2 + 3)(\alpha_3 + 2)$~~

~~$\alpha_1 = 1, \alpha_2 = 3, \alpha_3 = 2$.~~

$\therefore (r+1)(r+3)(r-2)$

If $n=0$; $a_0 = \alpha_1 (1)^0 + \alpha_2 (3)^0 + \alpha_3 (-2)^0$

$a_0 = \alpha_1 + \alpha_2 + \alpha_3$

$\alpha_1 + \alpha_2 + \alpha_3 = 7 \longrightarrow (1)$

If $n=1$, $a_1 = \alpha_1 (1)^1 + \alpha_2 (3)^1 + \alpha_3 (-2)^1$

$-4 = \alpha_1 + 3\alpha_2 - 2\alpha_3 \longrightarrow (2)$

If $n=2$, $a_2 = \alpha_1 (1)^2 + \alpha_2 (3)^2 + \alpha_3 (-2)^2$.

$8 = \alpha_1 + 9\alpha_2 + 4\alpha_3 \longrightarrow (3)$

from (1) & (2)

$\alpha_1 + \alpha_2 + \alpha_3 = 7$

$-\alpha_1 + 3\alpha_2 - 2\alpha_3 = -4$

$-2\alpha_2 + 3\alpha_3 = 11 \longrightarrow (4)$

from (2) & (3)

$\alpha_1 + 3\alpha_2 - 2\alpha_3 = -4$

$\alpha_1 + 9\alpha_2 + 4\alpha_3 = 8$

$-6\alpha_2 - 6\alpha_3 = -12 \longrightarrow (5)$

$-6(\alpha_2 + \alpha_3) = 2$

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from (4) & (5)

4 * (2) ⇒ $-4\alpha_2 - 6\alpha_3 = 22$

(5) ⇒ $-6\alpha_2 - 6\alpha_3 = -12$

$-10\alpha_2 = 10$

$\boxed{\alpha_2 = -1}$ Subs in (4)

$-2(-1) + 3\alpha_3 = 11$

$2 + 3\alpha_3 = 11$

$3\alpha_3 = 11 - 2$

$3\alpha_3 = 9$

$\boxed{\alpha_3 = 3}$

Subs α_2 & α_3 in (1)

$\alpha_1 + (-1) + 3 = 7$

$\alpha_1 = 7 - 2$

$\boxed{\alpha_1 = 5}$

$\therefore a_n (5)(1)^n + (-1)^n 3^n + 3(-2)^n$ ✓

(8) Find the solution to $a_n = 7a_{n-2} + 6a_{n-3}$ with $a_0 = 9$, $a_1 = 10$ and $a_2 = 32$

Sol

$a_n = 7a_{n-2} + 6a_{n-3}$:

~~$r^3 = 7r + 6$~~ $r^3 = 7r + 6$

~~$r^3 - 7r - 6 = 0$~~ $r^3 + 0.r^2 - 7r - 6 = 0$

$r^3 - 7r - 6 = 0$

$r=-1$	1	0	-7	-6
	0	-1	1	6
	1	-1	-6	0

$r = -1$ | $r^2 - r - 6 = 0$

$r^2 - 3r + 2r - 6 = 0$

$r(r-3) + 2(r-3) = 0$

$(r-3)(r+2) = 0$

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$\therefore r = -1, \; r_2 = 3, \; r_3 = -2.$

$a_n = (-1)^n \alpha_1 + (3)^n \alpha_2 + (-2)^n \alpha_3$

$a_0 = (-1)^0 \alpha_1 + (3)^0 \alpha_2 + (-2)^0 \alpha_3$

$\alpha_1 + \alpha_2 + \alpha_3 = 9 \rightarrow (1)$

$a_1 = -\alpha_1 + 3\alpha_2 - 2\alpha_3$

$\Rightarrow -\alpha_1 + 3\alpha_2 - 2\alpha_3 = 10 \rightarrow (2)$

(90)

$a_2 = \alpha_1 + 9\alpha_2 + 4\alpha_3 = 32.$

$\alpha_1 + 9\alpha_2 + 4\alpha_3 = 32 \rightarrow (3)$

From (1) & (2)

$\alpha_1 + \alpha_2 + \alpha_3 = 9$

$-\alpha_1 - 3\alpha_2 - 2\alpha_3 = 10$

$-2\alpha_2 - \alpha_3 = 19 \rightarrow (4)$

From (2) & (3)

$-\alpha_1 + 3\alpha_2 - 2\alpha_3 = 10$

$\alpha_1 + 9\alpha_2 + 4\alpha_3 = 32$

$12\alpha_2 + 2\alpha_3 = 42 \rightarrow (5)$

From (4) & (5)

Equ 4 × 2 $\Rightarrow -8\alpha_2 - 4\alpha_3 = 56$

$12\alpha_2 - 4\alpha_3 = 42$

$-20\alpha_2 = 14$

$\alpha_2 = -7/10.$

$-2\alpha_2 - \alpha_3 = 19$

$-2\left(\frac{-7}{10}\right) - \alpha_3 = 19$

$-\alpha_3 = 19 - 7/5$

$-\alpha_3 = \frac{95-7}{5}$

$-\alpha_3 = \frac{88}{5}$

$a_3 = -\frac{88}{5}.$

$\alpha_1 + \alpha_2 + \alpha_3 = 9$

$\alpha_1 - \frac{7}{10} - \frac{88}{5} = 9$

$\alpha_1 - \frac{7-176}{10} = 9$

$\alpha_1 + \frac{-183}{10} = 9$

$\alpha_1 = 9 + \frac{183}{10}$

$\alpha_1 = \frac{90+183}{10} = \frac{273}{10.}$

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24th NOV

Generating functions :—

The generating function for the sequence $a_0, a_1, \ldots, a_k, \ldots$ of real numbers is the infinite series $G(x) = a_0 + a_1 x + \ldots + a_k x^k + \ldots = \sum_{k=0}^{\infty} a_k x^k$.

Note:- The Generating function for $\{a_k\}$ given in definition Put $k=1$, then it is called ordinary generating function of $\{a_k\}$.

(91)

Problem:-

1) The generating function for the sequence $\{a_k\}$ with $a_k = 3$, $a_k = k+1$ and $a_k = 2^k$.

Sol:- G.F $a_k = k+1$, $a_k = 2^k$, $a_k = 3$

By the defination of generating function

$$g(x) = \sum_{k=0}^{\infty} a^k \cdot x^k.$$

∴ generating functions are $= \sum_{k=0}^{\infty} 3 \cdot x^k.$

$$= \sum_{k=0}^{\infty} (k+1)\, x^k.$$

$$= \sum_{k=0}^{\infty} 2^k \cdot x^k.$$

1. Let $\sum_{k=0}^{\infty} 3 \cdot x^k$:

$= 3 \cdot x^0 + 3 \cdot x^1 + \ldots$

$= 3 + 3x + \ldots$

2. $\sum_{k=0}^{\infty} (k+1)\, x^k.$

$= (k+1)\, x^0 + (k+1)\, x^1 + \ldots$

$= (0+1) + (1+1)x + \ldots$

$= 1 + 2x + \ldots$

$= 1 + 2x.$

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3. $\sum_{k=0}^{\infty} 2^k \cdot x^k$.

$= 2^0 \cdot x^0 + 2^1 x^1 + \cdots$

$= x + 2x + \cdots$

$= 3x$ //

2) what is the generating function for the sequence. 1, 1, 1, 1, 1 ?

Sol: $a_0 = 1,\ a_1 = 1,\ a_2 = 1,\ a_3 = 1,\ a_4 = 1,\ a_5 = 1$.

Therefore the generating function is

$$G(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5.$$

$$\therefore G(x) = 1 + x + x^2 + x^3 + x^4 + x^5.$$

Let us suppose that $1 + x + x^2 + x^3 + x^4 + x^5 = \frac{x^6 - 1}{x - 1}$ //

3) Let m be a the integer., Let $a^k = c(m,k)$, $k = 0,1,2,\ldots$ m., what is the generating function for the sequence. $a_0, a_1, a_2 \ldots a_m$.

Sol:

G.F $a_k = c(m,k)$

$a_0 = c(m,0)$

$a_1 = c(m,1)$

$a_2 = c(m,2)$

⋮

$a_m = c(m,m)$

∴ Generating function $G(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots a_k x^k$

$= c(m,0) + c(m,1)x + c(m,2)x^2 + \cdots + c(m,m)x^m$ //

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4) find the Generating function for the finite seq 1, 4, 16, 64, 256.

Sol:- $a_0 = 1,\ a_1 = 4,\ a_2 = 16,\ a_3 = 64,\ a_4 = 256.$

$G(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4.$

$= 1 + 4(x) + 16x^2 + 64x^3 + 256x^4.$ (93)

$= 1 + 4x + 16x^2 + 64x^3 + 256x^4$ /—

5) find a closed form for the G.F for the sequence.

$a_n = 5,\ n = 0, 1, 2 \text{---}$

Sol:- Given that $a_n = 5.$

$a_0 = 5,\ a_1 = 5,\ a_2 = 5 \text{-----}$

there fore G.F $G(x) = 5 + 5x + 5x^2 + \text{----}$ /—

6) find the G.F for the sequence $a_n = 2$, for $n = 3, 4, 5 \text{---}$ and $a_0 = a_1 = a_2 = 0.$

Sol:- Given $a_n = 2$

$a_0 = a_1 = a_2 = 0.$

$a_3 = 2,\ a_4 = 2,\ a_5 = 5.$

∴ G.F $G(x) = 0 + 0 + 0 + 2x^3 + 2x^4 + 2x^5 + \text{----}$ /—

Binomial Extended Co-efficients. (8) Extended Binomial Co-efficients :—

Let u be a real number. and k be a non-negative integer. then the Extended binomial co-efficients. $\binom{u}{k}$ is defined by. $\binom{u}{k} = \begin{cases} u(u-1)(u-2)\cdots(u-k+1)/k; & \text{if } k>0 \\ 1 & ; \text{if } k=0 \end{cases}$

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Problem :-

1) find the values of the Extended binomial Co-efficients

~~$(-2,3)$ & $(\frac{1}{2},3)$~~ $\binom{-2}{3}$ & $\binom{1/2}{3}$

Sol:- 1. Given that $\binom{-2}{3} = \binom{u}{k}$

$u = -2, \quad k = 3 > 0$

$$\binom{u}{k} = u(u-1)(u-2)\cdots(u-k+1)/k!$$

$$\binom{-2}{3} = \frac{(-2)(-2-1)(-2-2)\cdots(-2-3+1)}{3!}$$

$$= \frac{(-2)(-3)(-4)}{3\times 2\times 1} = -4/-$$

2. $\binom{1/2}{3} = \binom{u}{k}$

$u = 1/2, \quad k = 3 > 0.$

$$\binom{u}{k} = u(u-1)(u-2)\cdots(u-k+1)/k!$$

$$\binom{\frac{1}{2}}{3} = \frac{(\frac{1}{2})(\frac{1}{2}-1)(\frac{1}{2}-2)\cdots(\frac{1}{2}-3+1)}{3!}$$

$$= \frac{(\frac{1}{2})(-\frac{1}{2})(\frac{-3}{2})}{3\times 2\times 1} = \frac{\frac{3}{8}}{6} = \frac{3}{8}\times\frac{1}{6_2} = 1/16/-$$

Theorem:- The Extended binomial theorem :-

Let 'x' be a real number with $|x|<1$ & let u be a real number then $(1+x)^u = \sum_{k=0}^{\infty}\binom{u}{k}x^k$.

Proof:- This theorem is proved by mecloriaus. Mecloriaus theorem

$f(x) = (1+x)^u \Rightarrow f(x) = (1+0)^u = 1$

$f'(x) = u(1+x)^{u-1} \Rightarrow f'(0) = u.$

$f''(x) = u(u-1)(1+x)^{u-2} = u(u-1)$

$f'''(x) = u(u-1)(u-2)(1+x)^{u-3} = u(u-1)(u-2)$

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By the maclorous theorem

$$a(x) = a(0) + \frac{a'(0)}{1!} + \frac{a''(0)}{2!} + \frac{a'''(0)}{3!} + \cdots\cdots$$

$$= 1 + \frac{u}{1!} + \frac{u(u-1)}{2!} + \cdots\cdots$$

$$= 1 + u\cdot x + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + \cdots\cdots$$

$$= \sum_{k=0}^{\infty} \binom{u}{k} x^k \text{ //-}$$

Problems:-

1) find the Generating functions for $(1+x)^{-n}$ and $(1-x)^{-n}$. with 'n' is +ve integer using the Extended binomial theorem.

Proof:- we know that the Extended binomial theorem.

$$(1+x)^u = \sum_{k=0}^{\infty} \binom{u}{k} x^k.$$

Put $u = -n$

$$\therefore (1+x)^{-n} = \sum_{k=0}^{\infty} \binom{-n}{k} x^k.$$

Put $u = -n$ and $x = -x$, then we get

$$(1-x)^{-n} = \sum_{k=0}^{\infty} \binom{-n}{k} (-x)^k \text{ //-}$$

Expantional Generating function:-

The Power series of permutation of 3-elements a, b, c. we get $1 + (a+b+c)x + (ab+bc+ca+ba+cb+ac)x^2 + (abc+acb+bac+bca+cab+cba)x^3$

but this polynomial is Equivalent to $1 + (a+b+c)x + 2(ab+bc+ca)x^2 + 6(abc)x^3$

Above Equations can be Expressed in the form of

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$$F(x) = P(n,0)x^0 + P(n,1)x^1 + P(n,2)x^2 + \cdots + P(n,n)x^n$$

$$= 1 + \frac{n!}{(n-1)!}x + \frac{n!}{(n-2)!}x^2 + \cdots + \frac{n!}{(n-n)!}x^n$$

∴ unfortunately there is no simple closed form expression for the above function.

we know that $(1+x)^n = 1 + \frac{n!}{(n-1)!1!}x + \frac{n!}{(n-2)!2!}x^2 + \cdots + x^n$.

$$= 1 + \frac{P(n,1)}{1!}x + \frac{P(n,2)}{2!}x^2 + \cdots + x^n.$$

By the expantional Generating function

Let $a_0, a_1, a_2 \cdots a_n$ be a sequence then the function

$$F(x) = a_0 + \frac{a_1}{1!}x + \frac{a_2}{2!}x^2 + \cdots + \frac{a_r}{r!}x^r + \cdots$$

The above Expansional is called Expantional Generating function/—

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UNIT - IV

Graph Theory.

Graph :- A Graph 'G' is a pair of sets (V, E) where, 'V' is non-empty set. the set 'V' is called the set of vertices (points) and the set E is called set of Edges (lines) i.e; G = (V, E)

Types of a Graph :-

1. **Directed graph :-** Let G = (V, E) be a graph if the Elements of E are (countable set) ~~pairs~~ ordered pairs of vertices. then the graph G is called directed graph. Suppose if $(u, v) \in E$ is said to be from u to v.

   Indirected graph :- In a graph G = (V, E) if the Elements of E are unordered. (uncountable set) pair of vertices then the graph G is called undirected graph. Suppose if $(u, v) \in E$ is said to be from u to v.

   Note :- In a graph G = (V, E) the set of vertices is denoted by V(G) or V and the set of edges of the graph is denoted by E(G) or E.

   ⇒ |V(G)| = No of vertices of a graph G and also called Order of the graph G.

   ⇒ |E(G)| = No. of Edges of graph G and also Called as Size of the graph 'G'.

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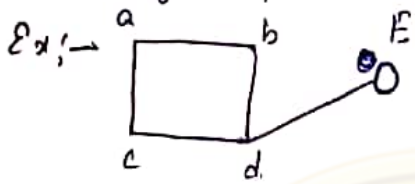
Direct graph :-

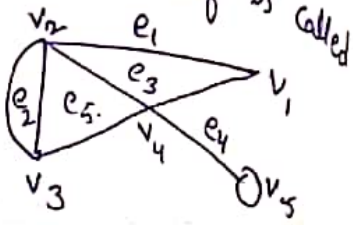
finite graph :- A graph $G = (V, E)$ is said to be finite if V and E are finite.

Self loop :- A self loop is an edge instant and vertex to itself. (or)

An Edge which is b/w a vertex and itself is called a self loop.

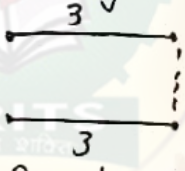
Ex :-



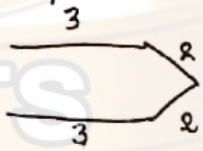


In the above graph is a self loop instant and vertex of E.

Parallel Graph :- The Edges which have same End vertices are called Parallel Edges.

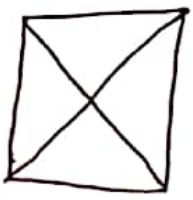


Simple graph :- A Graph $G = (V, E)$ is said to be simple if there no self-graph and no Parallel Edges

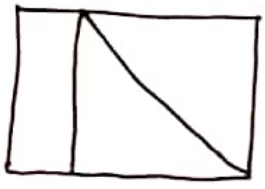


Multi graph :- A Graph $G = (V, E)$ is said to Multi graph if there is atleast one Edge b/w a pair of vertices.

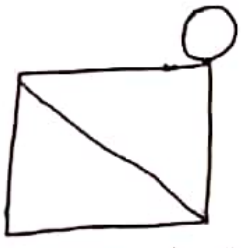
Ex :-



Simple graph



Multi graph.



Either Simple (or) Multi graph.

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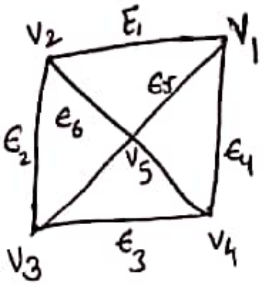
Order and Size of graph:-

The no. of vertices in a graph is called the order of the graph. The no. of Edges in a graph is called the Size of graph.

Ex:-



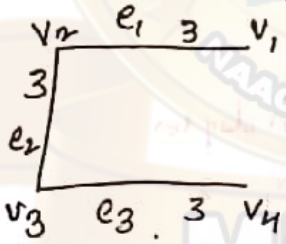
Graph = $G(4,6)$
Order = 4
Size = 6.

Adjacent Edges:- Two non-parallel Edges which are incident on a common vertex are called Adjacent Edges.

Ex:-

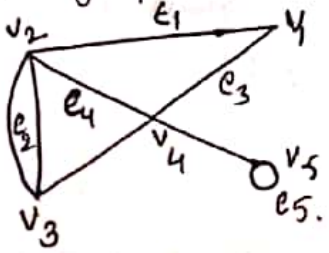
Adjacent vertices:- Two vertices which have an edge joining are called adjacent vertices.

Ex:-



Degree of a vertex:- the no. of Edges instant on a vertex of a graph. $G = (V, E)$ with self-loop counted as twice called degree of a vertex.

General graph:- A graph is said to be general graph if it is not a simple graph.



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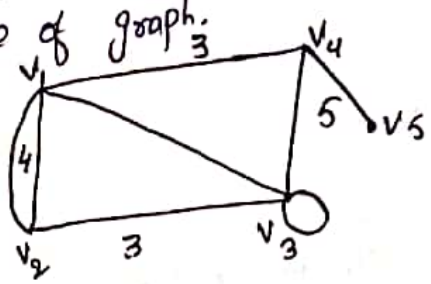
Degree of a graph :— The sum of all vertices is called a degree of graph.

Ex :—



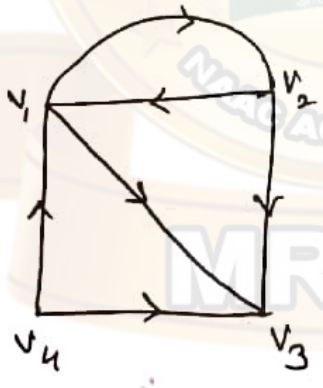
Degree of vertices $v_1 = 4$, $v_2 = 3$, $v_3 = 5$, $v_4 = 3$, $v_5 = 1$

degree of graph $\sum_{i=1}^{5} \deg v_i^\circ = 16$.

Indegree and outdegree of a directed graph :—

In a directed graph 'G' the no. of Edges instant to vertex v_i is called Indegree of graph G and it is denoted by $\deg' v_i'$. In a graph G the no. of Edges instant from vertex 'v_i' is called out degree of graph G. and it is denoted by $\deg_G^- v_i$.

Ex :—



In degree	Out degree
$v_1 = 2$	$v_1 = 2$
$v_2 = 1$	$v_2 = 1$
$v_3 = 0$	$v_3 = 0$
$v_4 = 2$	$v_4 = 2$
$\sum_{i=1}^{4} \deg_{G^+} v_i^\circ = 5$	$\sum_{i=1}^{4} \deg_{G^-} v_i^\circ = 5$

Theorem :—

The sum of degrees of all vertices is any non-directed is Equals to twice the no. of edges $|V|$.

i.e., $\sum_{i=1}^{|V|} \deg v_i^\circ = 2|E|$.

Proof :— Let $G = (V, E)$ be a non-directed graph.

and Let $V = \{v_1, v_2 \ldots\ldots v_n\}$ be the vertex set.

$(\because |V| = n)$

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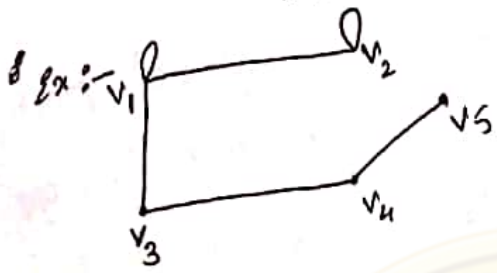
since Each Edge contributes to the sum of left side as well as right side.

⇒ The sum of degrees of all vertices Equal to twice the no. of Edges.

∴ Hence $\sum_{i=1}^{|V|=n} \deg v_i = 2|E|$.

Ex :-



degree :- $v_1 = 5$, $v_2 = 3$, $v_3 = 2$, $v_4 = 3$, $v_5 = 1$.

Total degree : 14 (5+3+2+3+1 = 14).

Clearly $\sum_{i=1}^{5} \deg v_i = 2 \times 7 = 14$. /—

Theorem :- The no. of odd degree vertices in any non-directed graph is always even.

Proof :- Let $G = (V, E)$ be any non-directed graph.

Let $v_1, v_2 \text{-----} v_k$ be a set of odd degree vertices.

and Let $w_1, w_2 \text{-----} w_k$ be the set of Even degree vertices.

i.e., $\sum_{i=1}^{n} \deg w_i = \text{even}$. → ①

we know that $\sum_{i=1}^{|V|=n} \deg \text{ of } v_i = 2 \times |E| = \text{Even}$ → ②

Now $\sum_{i=1}^{|V|=n} \deg v_i = \sum_{i=1}^{k} \deg v_i + \sum_{i=1}^{M} \deg w_i$.

$$2|E| = \sum_{i=1}^{k} \deg v_i + \text{Even.} \quad [① \& ②]$$

$$\text{Even} = \sum_{i=1}^{k} \deg v_i + \text{Even.}$$

$\sum_{i=1}^{k} \deg v_i$ = must be Even.

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there is possible when no. of odd degree vertices must be Even /—

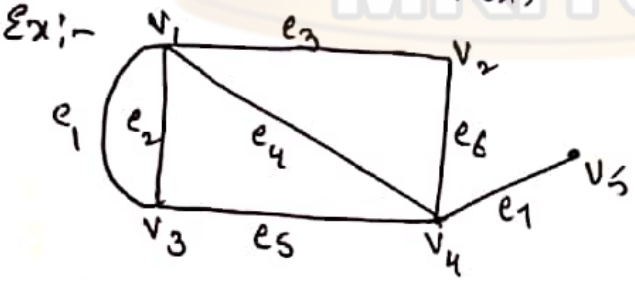
Theorem :— In a directed graph. $\sum_{i=1}^{|V|} \deg_G^+ V_i = \sum_{i=1}^{|V|} \deg_G^- V_i = |E|$

Proof :— Let $G = (V, E)$ be a simple directed graph. Each edge is directed and no more than one directed edge b/w the pair of vertices. Each edge accounts from one indegree and one outdegree.

⇒ the sum of indegree of all vertices equals to sum of outdegree of all vertices and both are equal to no. of edges.

Adjacently of vertices :— In a graph G. two non-parallel edges are said to be adjacent if there are instant of same vertex.

Ex :-



$\{v_1, v_2\}$, $\{v_1, v_3\}$, $\{v_1, v_4\}$ ----- are adjacent of vertices.

$\{v_1, v_5\}$ $\{v_3, v_5\}$ ---- are non-adjacent vertices /

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Adjacency of Edges:—

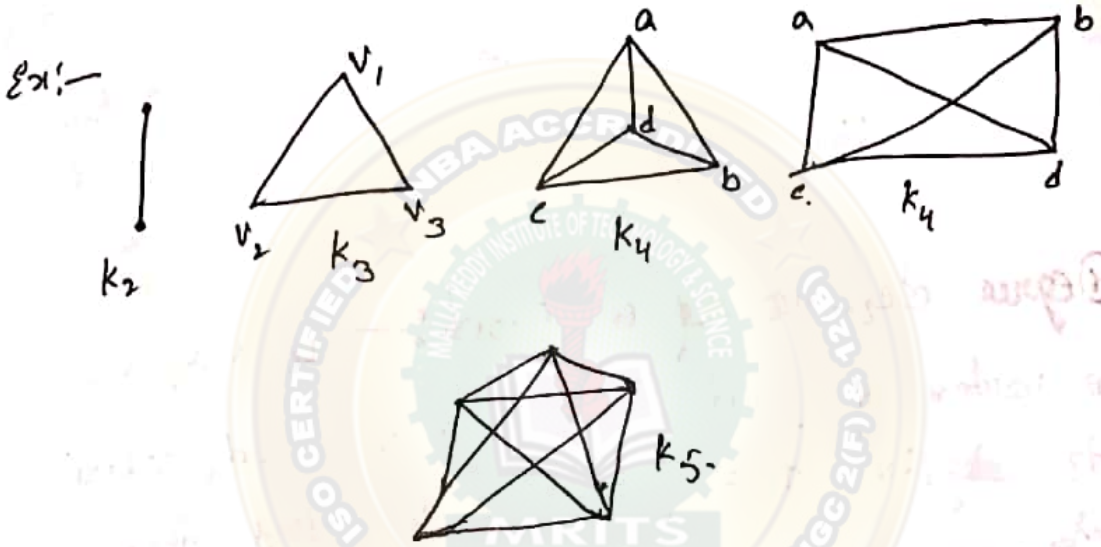
$\{e_1, e_2\}, \{e_2, e_3\}, \{e_3, e_6\}$ ---- are adjacent Edges.

$\{e_3, e_7\}, \{e_3, e_5\}$ ---- are non-adjacent edges.

Complete Graph:— A Graph $G = (V, E)$ is said to be complete if every two vertices in terms of adjacents

A complete graph of orders 'n' is denoted by 'K_n'.

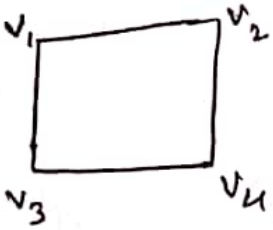
Ex:—



K-Regular graph:— A graph G is said to be k-regular graph if every vertex of degree k.

Note:— Every complete graph is k-regular graph. but converse need not be true.

Ex:—



The above graph G is regular graph but not complete, since the pairs $\{v_1, v_3\}, \{v_2, v_4\}$ are non-adjacent.

→ In, K_n graph every vertex of degree is equal to $n-1$.

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Problems:—

1) P.T In K_n, $|E| = \frac{n(n-1)}{2}$ where $|V| = n$.

Sol:- In K_n - graph we know that each vertex of degree is $n-1$ and also we know that.

$$\sum_{i=1}^{|V|} \deg v_i = 2|E|$$

$$\Rightarrow \sum_{i=1}^{|V|=n} (n-1) = 2|E|$$

$$n(n-1) = 2|E|$$

$$|E| = \frac{n(n-1)}{2}$$ /—

Degree Sequence of a graph:— If $v_1, v_2, v_3 \ldots v_n$ be vertex of graph G then $d_1 d_2 \ldots d_n$ where $d_i = \deg(v_i)$ of a graph G is called degree Sequence of a graph.

Note:— Degree Sequence is always monotonically increasing.

i.e., $\delta(G) \leq d_1 \leq d_2 \ldots \leq d_n \leq \Delta(G)$

where $\delta(G) = \min\{\deg v_i / v_i \in V\}$

$\Delta(G) = \max\{\deg v_i / v_i \in V\}$.

Isomorphism of a Graph:— Two graphs G and G' are said to be isomorphic if $\exists$ one-one correspondence b/w vertices of two graphs. which resolves the adjacency of vertices as well as the direction of Edges.

(if any) (one-one & onto).

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Necessary Conditions :—

1. No. of vertices of 1st graph G = No. of vertices of 2nd graph.

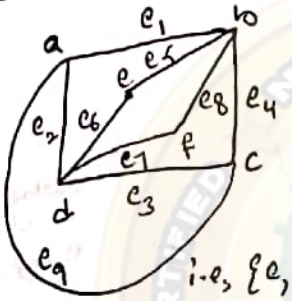
$$|V(G)| = |V(G')|$$

2. $|E(G)| = |E(G)'|$

3. degree sequence of 1st graph G = degree sequence of 2nd graph.

Problems :—

1) Such that whether the following graph are Isomorphic or not?



i.e., {e, f}

i.e., {a', f'}

Sol:- Both graphs contains same no. of vertices.

i.e; $|V(G)| = |V(G')| = 6$

$|E(G)| = |E(G')| = 9.$

Both graphs contains same no. of Edges.

i.e; $|E(G)| = |E(G')| = 9$

degree sequence of 1st graph G = degree sequence of 2nd graph.

$G = G' = \{3, 4, 3, 4, 2, 2\}$

Consider the mapping $f: V(G) \rightarrow V(G')$

degree of graph G	degree of graph G'.
a = 3	a' = 2
b = 4	b' = 3
c = 3	c' = 4
d = 4	d' = 3
e = 2	e' = 4
f = 2	

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Clearly f is one-one and onto.

$\therefore G \cong G'$ /—

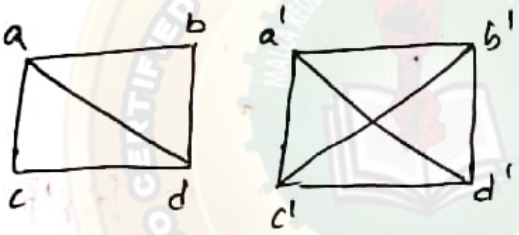
Verification of Adjencency :—

Vertex of degree two pairs is 1st graph non-adjacency adjacency.

Similarly in 2nd graph vertex of degree two pairs non-adjacent. but vertex of 3 pairs in 1st graph {a,c} adjacent and non-adjacent in 2nd graph {b', d'}

i.e. f does not preserve the adjacent.

Here. $G \ncong G'$.

2) 

whether the following graph are isomorphic or not.

Sol:- degree of G

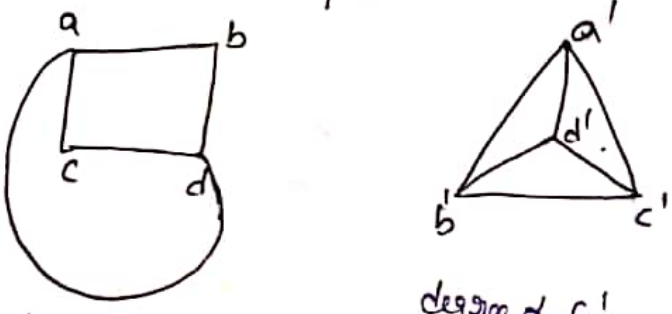
a = 3
b = 2
c = 2
d = 3

degree of G'

a' = 3
b' = 3
c' = 3
d' = 3

$\therefore$ It is not an isomorphic. (It is not satisfying the 2nd condition)

$\therefore G \ncong G'$ /—

3. 

Sol:- degree of G

a = 3, b = 2
c = 2, d = 3

degree of G'.

a = 3, b = 3
c = 3. d = 3

$\therefore G \ncong G'$ /—

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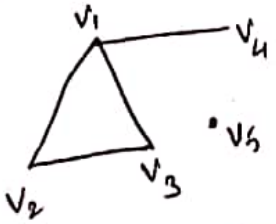
Isolated vertex:— A vertex of Graph G with zero degree is called Isolated vertex.

ex:-

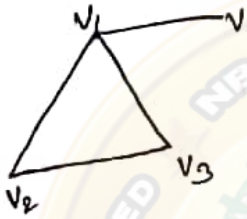


(degree $v_5 = 0$)

Isolated vertex $V = v_5$.

Pendent vertex:— A vertex of a graph G with degree one. is called pendent vertex.

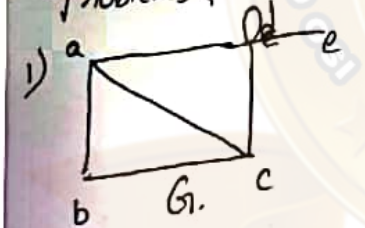
ex:-

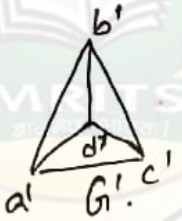


degree $v_4 = 1$ is pendent vertex.

Problems:—

1)





Sol:- Both graphs contains same no. of vertices.

i.e.,

$$|v(G)| = 5 = |v(G')|$$

Both graph contains same no. of Edges

i.e., $|E(G)| = 7 = |E(G')|$

degree sequence of 1st graph G = degree sequence of 2nd graph.

$\{3, 3, 2, 5, 1\}$

Consider the mapping $f: v(G) \to v(G')$

Degree of G	Degree of G'.
$a = 3$, $b = 2$, $c = 3$	$a' = 3$, $b' = 2$, $c' = 3$
$d = 5$, $e = 1$	$d' = 5$, $e' = 1$.

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Clearly f is one-one and onto.

Consider the mapping $f: V(G) \to V(G')$

Verification of adjacent :-

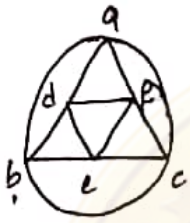
degree of 5 pair = {d}

degree of 2 pair = {b}

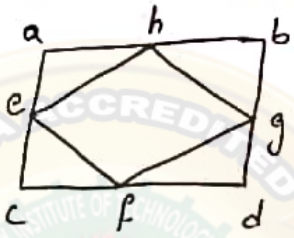
degree of 3 pair = {a, c}

∴ G is isomorphic to G' ⇒ G ≅ G' /—

2)



G

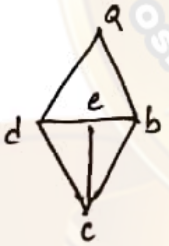


G'.

Sol:- $V(G) \neq V(G')$

∴ It is not isomorphic and Isolated /—

3)



Sol:- Both graph contains same no. of vertices

i.e., $V(G) = V(G') = 5$.

Both graph contains same no. of Edges.

i.e., $E(G) = E(G') = 6$

The degree of deg of 1st graph = degree deg of 2nd graph.

= {3, 3, 3, 2, 2}

Consider the mapping $f: V(G) \to V(G')$

degree of G	degree of G'
a = 2, b = 3, c = 3	a' = 3, b' = 3, c' = 3
d = 3, e = 3	d' = 2, e' = 3

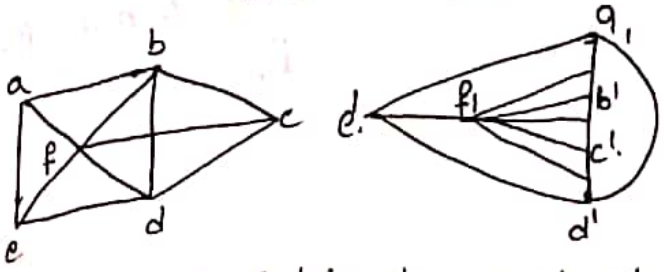
∴ f is one-one and onto

verification of adjacent :—

degree of 3 = {b, c, d}

degree of 2 = {a, e}

$G \cong G'$. /—

4) 

Sol:- Both graph contain same no. of vertices.

i.e: $V(G) = V(G') = 6$

Both graph contain same no. of Edges

i.e, $E(G) = E(G') = 12$.

The degree of sequence of 1st graph = degree of sequence of 2nd graph

$= \{3, 4, 3, 4, 3, 5\}$

Consider the mapping $f: V(G) \to V(G')$

degree of graph G	degree of graph G'
a = 3, b = 4, c = 3	a' = 4, b' = 3, c' = 3
d = 4, e = 3, f = 5.	d' = 4, e' = 3, f' = 5.

Clearly f is one-one and onto.

verification of adjacent :—

degree of 5 pairs = {f}

degree of 4 pairs = {b, d}

degree of 3 pairs = {a, c, e} /—

5) How many vertices the graph contains if

1. 16 edges and all vertices of degree 2.

Sol:- Given no. of Edges = 16

degree of each vertex = 2.

$\sum_{i=1}^{|V|} \deg v_i = 2|E|$

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$$\sum_{i=1}^{|V|} 2 = 2 \times 16$$

$$2|V| = 32$$

$$|V| = 16 /-$$

(ii) 21 Edges and three vertices of degree 4 and other of degree 3. find no. of vertices.

Sol:- Given no. of edges = 21

no. of vertices of degree 4 = 3

$$\sum_{i=1}^{|V|} \deg v_i + \sum_{i=4}^{|V|} \deg v_i = 2|E| = 2 \times 21.$$

$$\sum_{i=1}^{3} 4 + \sum_{i=1}^{|V|} 3 = 42$$

$$4 \times 3 + 3(|V| - 3) = 42$$

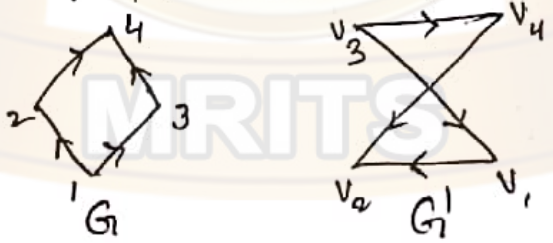
$$12 + 3|V| - 9 = 42$$

$$3|V| = 42 + 9 - 12$$

$$3|V| = 39.$$

Total no. of vertices are 13 and no. of vertices of degree 3 = 10 /-

6). S.T the graphs are isomorphic.



Sol:- No. of vertices of G = 4.

No. of vertices of G' = 4.

→ Both graph contains same no. of vertices

Indegree Sum of G : 0+1+2+1 = 4

Indegree Sum of G' = 1+2+0+1 = 4

Indegree of Sum of G = Indegree Sum of G'.

out degree Sum of G = 2+1+1+0 = 4.

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out degree sum of $G' = 1+0+2+1 = 4$.

out degree sum of G = out degree sum of G'.

∴ degree seq of G and G' are same.

Consider a function $f: v(G) \to v(G')$ such that degree spectrum of $G = \{$ Indegree, out degree $/ v \in V\}$

Degree spectrum $(G) = \{(0,2), (1,1), (1,1), (2,0)\}$

Degree spectrum $(G') = \{(1,1)(2,0)(0,2)(1,1)\}$

The vertices of degree spectrum (1,1) pair.

i.e., (2,3) in graph G not adjacent.

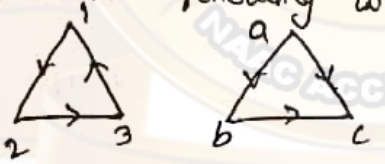
Similarly in the graph G', the vertices of degree spectrum (1,1) pair

i.e., (v_1, v_4) is also not adjacent. ∴ Preserves adjacently.

∴ $f: v(G) \to v(G')$ is an isomorphism.

∴ $G \cong G'$

7) Show that the following are not isomorphic.



Sol:- No. of vertices of graph $G = 3$

No. of vertices of graph $G' = 3$.

Both graph contains same no. of vertices.

Indegree of graph $G = 1+1+1 = 3$

Indegree of graph $G' = 0+1+2 = 3$.

∴ Indegree of G = Indegree of G'.

out degree of $G = 1+1+1 = 3$

out degree of $G' = 2+1+0 = 3$

∴ out degree of G = out degree sum of G'.

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∴ degree seq of G' and G are same.

Consider a function $f: V(G) \to V(G')$ such that

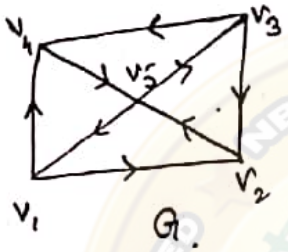
degree of spectrum $G = \{$ Indegree, outdegree $/ v \in V\}$

Degree spectrum $(G) = \{(1,1)\ (1,1)\ (1,1)\}$

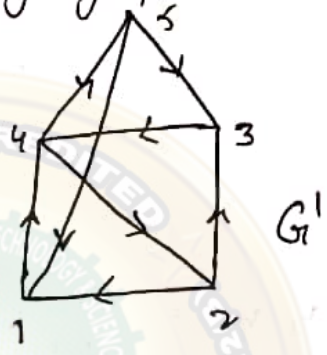
Degree spectrum $(G') = \{(0,1), (1,1)\ (1,0)\}$

There is no sequence of spectrum. /—

8. Show that the following graphs are Isomorphic.



G.



G'

Sol:— No. of vertices of $G = 5$.

No. of vertices of $G' = 5$.

both graphs contains same no. of vertices.

In degree of graph $G = 1+2+1+2+2 = 8$

Indegree of graph $G' = 2+1+2+2+1 = 8$

∴ Indegree of G = Indegree of G'.

out degree of graph $G = 2+1+2+1+2 = 8$

out degree of graph $G' = 1+2+1+2+2 = 8$

∴ out degree of graph G = outdegree of graph G'.

∴ deg of Sequence of G and G' are same.

Consider the function $f: V(G) \to V(G')$ such that

Spectrum of $D(G) = \{(1,2)\ (2,1)\ (1,2)\ (2,1)\ (2,2)\}$

Spectrum of $D(G') = \{(2,1)\ (1,2)\ (2,1)\ (2,2)\ (1,2)\}$

The vertex of degree spectrum $(1,2)$ pair.

i.e. (v_1, v_3) in the graph G are not adjacent.

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similarly the graph G' the vertex of degree spectrum (1,2) pair

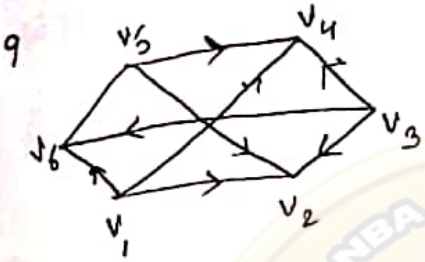
i.e. (2,5) are not adjacent.

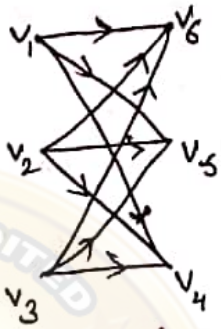
$\therefore f$ Preserves adjacenlty.

$\therefore f: V(G) \to V(G')$ is an isomorphic

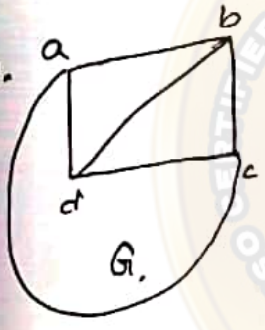
$\therefore G \cong G'$ /_

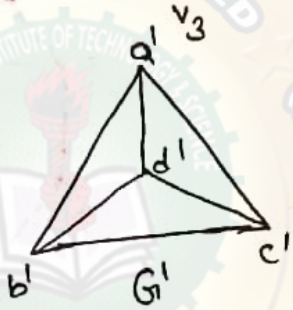
9





10.





Sol:- Both graphs contain same no. of vertices

i.e., $V(G) = V(G') = 4$

Both graphs contain same no. of edges

$|E(G)| = |E(G')| = 7$

The degree of sequence of 1st graph = degree sequence of 2nd graph.

$\{3,3,3,3\} = \{3,3,3,3\}$

degree of graph G

$a = 3$
$b = 3$
$c = 3$
$d = 3$

degree of graph G'

$a' = 3, b' = 3$
$c' = 3, d' = 3$

define $G(V) \to G(V')$ by $\phi(a) = a', \phi(b) = b', \phi(c) = c'$.
$\phi(d) = d'$.

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$\{a,b\} \xrightarrow{f} \{a',b'\}$ $\quad$ $\{b,d\} \xrightarrow{f} \{b',d'\}$

$\{b,c\} \xrightarrow{f} \{b',c'\}$ $\quad$ $\{d,a\} \xrightarrow{f} \{d',a'\}$

$\{c,d\} \xrightarrow{f} \{c',d'\}$ $\quad$ $\{a,c\} \xrightarrow{f} \{a',c'\}$

$\{d,a\} \xrightarrow{f} \{d',a'\}$

The vertices of deg 3 are adjacency in the graph G and adjacency in the graph G'.

$\therefore f: G \to G'$ is a bijective adjacency preserving mapping.

$\therefore f$ is an isomorphism.

$\therefore G \cong G'$ //-

11.

Sol:- Both graphs contains same no. of vertices.

i.e., $V(G) = V(G') = 6$

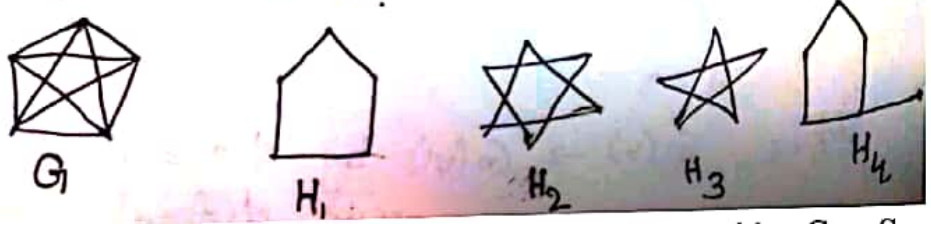
Both graph contains same no. of Edges

$E(G) \neq E(G')$.

$\therefore G \cong G'$ //-

Sub graph :- Let G, H be two non-Empty set. H is called subgroup of graph G. If V(H) is subset of V(G) and $V(H) \subseteq V(G)$, $E(H) \subseteq E(G)$ i.e., All the vertices of H are in G and all the Edges of H are in G.

Ex:-



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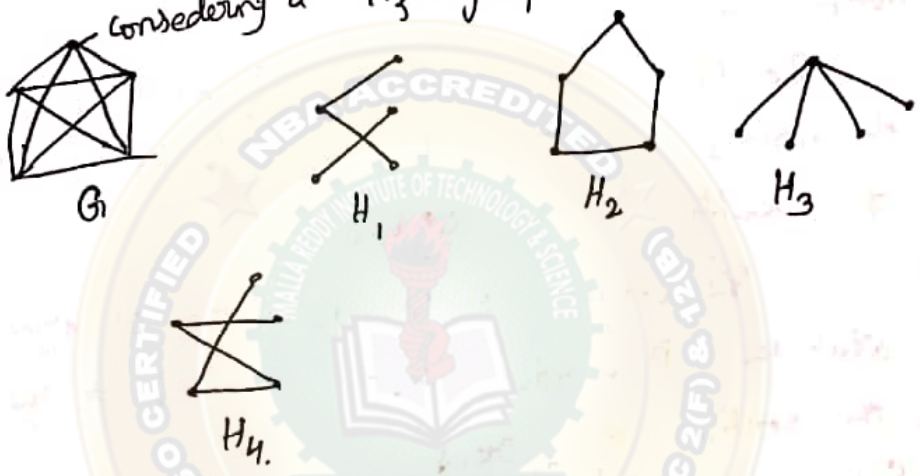
H_1, H_3 are subgraphs of G and

H_2, H_4 are not a subgraphs of G.

$V(H_2) \not\subset V(G)$, $V(H_4) \not\subset V(G)$

Complement of graph :— If H is a subgraph of graph G, then the complement of H in G is the graph obtained by deleting the edges of H from those of G.

Ex:— Considering a K_5 - graph

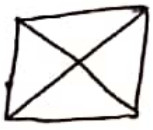


The complements of subgraph H_1, H_2, H_3 and H_4 are shown below.

$\bar{H}_1(G)$ =

$\bar{H}_2(G)$ =

$\bar{H}_3(G)$ =



,

$\bar{H}_4(G)$

Note :— The complement of H in G is denoted by $\bar{H}(G)$.

Multi - Graph :— A directed Multi - Graph can be represented as a set of mark points V. with set of arrows E b/w the points where there is a restriction on the no. of arrows from one point to another point.

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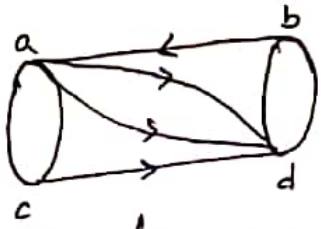
Ex:-



weighted graph :- we defined weighted graph as a ordered quadrible (V, E, F, G) are an ordered triple (V, E, F) are ordered triple (V, E, G) where 'V' is the set of vertices. E is set of Edges. F is a function whose domain is V and G is a function whose domain is E.

The function f is an assignment of weights to the vertices and the function. G is an assignment of weights to the edges.

The weight can be numbers or symbol or what the quantities that we wish to the assign to the vertices and edges.

Ex:-

(V, E, F, g) (V, E, F)

Path :- A path in directed graph is said to trivial tree. the nodes appear in the sequence originating in the initial node of first edge. and ending in the terminal node. of the last edge in the Equation.

(or)

Let G be a graph a sequence P of zero or more edges of the form $\{v_0\}, \{v_1, v_2\} \dots \{v_{n-1}, v_n\}$ is called Path v_0 to v_n it may be written as $v_1 - v_2 - v_3 \dots v_{n-1} - v_n$.

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Shortest Paths in a weighted graphs :—

Let $G = (V, E, w)$ be a weighted graph where w is a weighted function from E to R^+. i.e., $w : E \to R^+$. the weight of the edge i, j denoted by w_{ij} or $w(i, j)$ and is called the length of the edge i, j (or) shortest Path of graphs.

Ex:- Consider V is the set of cities and E is set of highways ~~connecting~~ covering the cities.

$w(i,j)$ = the weight of Edges (i,j)

= length of the edges (i, j)

= the distance b/w two cities (i, j)

Length of the path in G is defined to be the sum of the lengths of the edges in a path.

Dijkstra's algorithm :— For the shortest distance from a vertex 'a' to any vertex in $G = (V, E, w)$

Suppose that we have to determine a shortest Path from a vertex a to any vertex in G. Algorithm is in three steps.

1. Initially Let $P = \{a\}$

$T = V - \{a\}$

$= V - P$ for Every vertex $t \in T$.

Let $L(T) = w(a, T)$, $L(T) = \infty$ if a & t are not adjacent.

2. Select the vertex in T that has the smallest index with respect to P.

Let x denotes this vertex.

3. P [If x is the vertex in T that has the smallest index with respect to P.] x

3. If x is the vertex we wish to form a then stop If not $P' = P \cup \{x\}$ and $T' = T - \{x\}$ for Every $t \in T'$.

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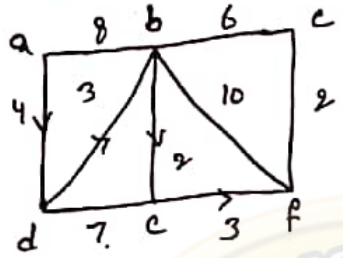
Compute the index w.r.to P according to the following formula.

$$l'(t) = \min \{ l(t), l(x) + \omega(x, t) \}$$

4. Repeat step 2 and 3 using P' as P and T' as T.

Problems:—

1) Find a minimum path from a to f.



Sol:— Let $P = \{a\}$

$T = V - \{a\}$

$T = \{a, b, c, d, e, f\} - \{a\}$

$T = \{b, c, d, e, f\}$

Let $l(b) = \omega(a, b) = 8$

$l(c) = \omega(a, c) = \infty$ (∵ a, e are not adjecent to each other)

$l(d) = \omega(a, d) = 4$,

$l(e) = \omega(a, e) = \infty$

$l(f) = \omega(a, f) = \infty$

Let $x = d$.

Let $P' = P \cup \{x\}$

$P' = \{a\} \cup \{d\}$

$P' = \{(a, d)\}$

$T' = T - \{x\}$

$T' = \{b, c, d, e, f\} - \{d\}$

$T' = \{b, c, e, f\}$

$l'(b) = \min \{ l(b), l(d) + \omega(d, b) \}$

$= \min \{8, 4+3\}$

$= \min \{8, 7\}$

$l'(b) = 7$

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$l'(d) = \min\{l(d), l(d) + w(d,d)\}$

$l'(c) = \min\{\infty, 4+\infty\}$

$= \min\{\infty, \infty\}$

$l'(c) = \infty$

$l'(e) = \min\{l(e), l(d) + w(d,e)\}$

$= \min\{\infty, 4+7\}$

$= \min\{\infty, 11\}$

$l'(e) = 11$

$l'(f) = \min\{l(f), l(d) + w(d,f)\}$

$= \min\{\infty, \infty + \infty\}$

$= \min\{\infty, \infty\}$

$l'(f) = \infty$

Let $x = b$ $\quad$ x': smallest index in $l'(b), l'(c), l'(e), l'(f)\}$

Let $P' = P \cup \{x\}$

$P' = \{a, d\} \cup \{b\}$

$P' = \{a, b, d\}$.

$T' = T - \{x\}$

$= \{b, c, e, f\} - \{b\}$.

$= \{c, e, f\}$

$l'(c) = \min\{l(c), l(b) + w(b,c)\}$

$= \min\{\infty, 7+6\}$

$= \min\{\infty, 13\}$

$l'(c) = 13$

$l'(e) = \min\{l(e), l(b) + w(b,e)\}$

$= \min\{11, 7+2\}$

$= \min\{11, 9\}$

$l'(e) = 9$.

$l'(f) = \min\{l(f), l(b) + w(b,f)\}$

$= \min\{\infty, 7+10\}$

$= \min\{\infty, 17\}$

$l'(f) = 17$.

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Let $x = e$ (∵ smallest index $l'(c), l'(e), l'(f)$)

Let $P' = P \cup \{x\}$

$P' = \{a, d, b\} \cup \{e\}$

$= \{a, d, b, e\}$

$T' = T - \{x\}$

$= \{c, e, f\} - \{e\}$

$= \{c, f\}$

$l'(c) = \min \{l(c), l(e) + \omega(e, c)\}$

$= \min \{\infty, 9 + \infty\}$

$= \min \{\infty, \infty\}$

$l'(c) = \infty$

$l'(f) = \min \{l(f), l(e) + \omega(e, f)\}$

$= \min \{\infty, 9 + 3\}$

$= \min \{\infty, 12\}$

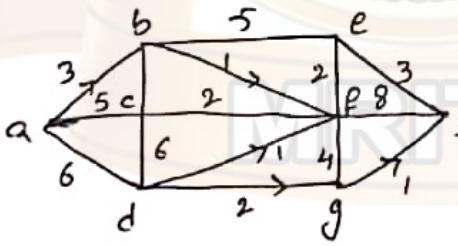
$l'(f) = 12.$

∴ min path is (a, d, b, e, f)

minimum distance is shortest distance from a to f.

$= 4 + 3 + 2 + 3 = 12$ /—

2).



Sol:- Let $P = \{a\}$

$T = V - \{a\}$

$T = \{a, b, c, d, e, f, g, z\} - \{a\}$

$T = \{b, c, d, e, f, g, z\}$

Let $l(b) = \omega(a, b) = 3$

$l(c) = \omega(a, c) = 5$

$l(d) = \omega(a, d) = 6$

$l(e) = \omega(a, e) = \infty$

$l(f) = \omega(a, f) = \infty$

$l(g) = \omega(a, g) = \infty$

$l(z) = \omega(a, z) = \infty$

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Let $x = b$ [∵ smallest index is 3]

Let $P' = P \cup \{x\} = \{a\} \cup \{b\} = \{a, b\}$

$T' = T - \{x\} = \{b, c, d, e, f, g, z\} - \{b\}$

$T' = \{c, d, e, f, g, z\}$

$l'(c) = \min\{l(c), l(b) + w(b, c)\}$; $l'(d) = \min\{l(d), l(b) + w(b, d)\}$

$l'(c) = \min\{5, 3+5\}$

$l'(c) = \min\{5, 8\}$

$l'(c) = 5.$

$= \min\{6, 6+\infty\}$

$= \min\{\infty, 6, \infty\}$

$l'(d) = 6$

$l'(e) = \min\{l(e), l(b) + w(b, e)\}$; $l'(f) = \min\{l(f), l(b) + w(b, f)\}$

$= \min\{\infty, 3+5\}$

$= \min\{\infty, 8\}$

$l'(e) = 8$

$= \min\{\infty, 3+1\}$

$= \min\{\infty, 4\}$

$l'(f) = 4$

$l'(g) = \min\{l(g), l(b) + w(b, g)\}$; $l'(z) = \min\{l(e), l(b) + w(b, z)\}$

$= \min\{\infty, 3+\infty\}$

$= \min\{\infty, \infty\}$

$l'(g) = \infty$

$= \min\{\infty, 3+\infty\}$

$= \{\infty, \infty\}$

$l'(z) = \infty$

Let $x = f$.

Let $P' = P \cup \{x\}$

$P' = \{a, b\} \cup \{f\} = \{a, b, f\}$

$T' = T - \{x\} = \{f, b, c, d, e, g, z\} - \{f\} = \{b, c, d, e, g, z\}$

$l'(c) = \min\{l(c), l(f) + w(f, c)\}$; $l'(d) = \min\{l, (d), l(f) + w(f, d)\}$

$l'(c) = \min\{5, 4+2\}$

$= \min\{5, 6\}$

$l'(c) = 5$

$= \min\{6, 4+1\}$

$= \{6, 5\}$

$l'(d) = 5.$

$l'(e) = \min\{l(e), l(f) + w(f, e)\}$; $l'(g) = \min\{l(g), l(f) + w(f, g)\}$

$= \min\{\infty, 4+2\}$

$= \{\infty, 6\}$

$l'(e) = 6.$

$= \min\{\infty, 4+4\}$

$= \min\{\infty, 8\}$

$l'(g) = 8$

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$l'(z) = \min\{l(z), l(f) + w(f,z)\}$

$= \min\{\infty, 4+8\}$

$= \min\{\infty, 12\}$

$l'(8) = 12$

Let $x = d$

Let $p' = p \cup \{x\} = \{a,b,f\} \cup \{d\} = \{a, b, d, f\}$

$T' = T - \{x\} = \{c, d, e, g, z\} - \{d\} = \{c, e, g, z\}$

$l'(c) = \min\{l(c), l(d) + w(d,c)\}$; $l'(e) = \min\{l(e), l(d) + w(d,e)\}$

$= \min\{5, 5+6\}$ $= \min\{\infty, 5+\infty\}$

$= \min\{5, 11\}$ $= \min\{\infty, \infty\}$

$l'(c) = 5$. $l'(e) = \infty$

$l'(g) = \min\{l(g), l(d) + w(d,g)\}$; $l'(z) = \min\{l(z), l(d) + w(d,z)\}$

$= \min\{\infty, 5+2\}$ $= \min\{\infty, 5+\infty\}$

$= \min\{\infty, 7\}$ $= \min\{\infty, \infty\}$

$l'(g) = 7$ $l'(z) = \infty$

Reject c since all the values are infinity there is no shortest value.

Let $x = g$. [smallest index of z]

Let $p' = p \cup \{x\} = \{a,b,f,d,c\} \cup \{g\} = \{a,b,f,d,c,g\}$

$T' = T - \{x\} = \{e,g,z\} - \{g\} = \{e, z\}$

$l'(e) = \{l(e), l'(g) + w(g,e)\}$; $l'(z) = \min\{l(z), l(g), w(g,z)\}$

$= \min\{\infty, 7+\infty\}$ $= \min\{\infty, 7+1\}$

$= \min\{\infty, \infty\}$ $= \{\infty, 8\}$

$l'(e) = \infty$ $l'(z) = 8$.

$\therefore$ min path is $\{a,b,f,d,g,z\}$

$= 3+1+1+2+1 = 8$ //—

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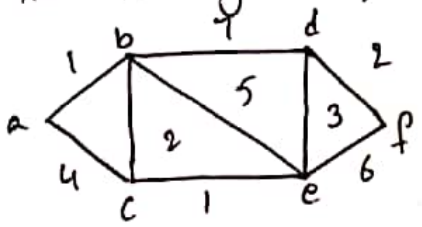
3. Apply Dijkstra's algorithm to determine a shortest path from a to f the following weighted graph.



Sol:- Let $P = \{a\}$

$T = V - \{a\} = \{a,b,c,d,e,f\} - \{a\} = \{b,c,d,e,f\}$

Let $l(b) = w(ab) = 1$ $\quad l(e) = w(a,e) = \infty$

$l(c) = w(a,c) = 4$. $\quad l(f) = w(a,f) = \infty$

$l(d) = w(a,d) = \infty$

Let $x = b$ ($\because$ smallest distance is 1}

Let $P' = P \cup \{x\} = \{a\} \cup \{b\} = \{a,b\}$

$T' = T - \{x\} = \{b,c,d,e,f\} - \{b\} = \{c,d,e,f\}$

$l'(c) = \min\{l(c), l(b) + w(b,c)\}$; $l'(d) = \min\{l(d), l(b) + w(b,d)$

$= \min\{4, 1+2\}$ $\Rightarrow \min\{\infty, \infty + 7\}$

$= \min\{4, 3\}$ $= \min\{\infty, 8\}$

$l'(c) = 3$ $\quad l'(d) = 8$

$l'(e) = \min\{l(e) + l(b) + w(b,e)\}$; $l'(f) = \min\{l(f), l(b) + w(b,f)\}$

$= \min\{\infty, 1+5\}$ $= \min\{\infty, 1+\infty\}$

$= \min\{\infty, 6\}$ $= \min\{\infty, \infty\}$

$l'(e) = 6$. $\quad l'(f) = \infty$

Let $x = c$;

Let $P' = P \cup \{x\} = \{a,b\} \cup \{c\} = \{a,b,c\}$

$T' = T - \{x\} = \{c,d,e,f\} - \{c\} = \{d,e,f\}$

$l'(d) = \min\{l(d), l(c) + w(c,d)\}$; $l'(e) = \min\{l(e), l(c) + w(c$

$= \min\{\infty, 3+\infty\}$ $= \min\{\infty, 3+1\}$

$= \min\{\infty, \infty\}$ $= \min\{\infty, 4\}$

$l'(d) = \infty$. $\quad l'(e) = 4$.

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$l'(f) = \min\{l(f), l(c) + w(c,f)\}$

$l'(f) = \min\{\infty, 3+\infty\}$

$= \min\{\infty, \infty\}$

$l'(f) = \infty$

Let $x = f$.

$P' = P' \cup \{x\} = \{a,b,c\} \cup \{e\} = \{a,b,c,e\}$

$T' = T - \{x\} = \{d,e,f\} - \{e\} = \{d,f\}$

$l'(d) = \min\{l(d), l(e) + w(d,e)\}$

$= \min\{\infty, 4+3\}$

$= \min\{\infty, 7\}$

$l'(d) = 7$

$l'(f) = \min\{l(f), l(e) + w(e,f)\}$

$= \min\{\infty, 4+6\}$

$= \min\{\infty, 10\}$

$l'(f) = 10.$

∴ minimum path is $\{a,b,c,d,e,f\}$

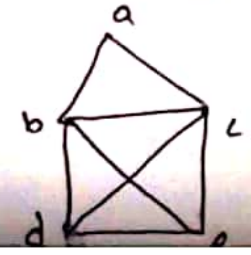
$= 1+2+1+3+2 = 9$ //—.

Eulerian Path and Eulerian circuits :—

A path is an undirected graph. $G = (V, E)$ is said to be Eulerian path if it transverses each edge in the graph once and only one.

A circuit is an undirected graph $G = (V,E)$ is said to be Eulerian circuit if it transverses each vertex in the graph. once and only once.

Ex :—



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It is an undirected graph it has the Eulerian Path. and the Eulerian Path d, b, a, c, b, e, c, d, e. It has no. Eulerian circuits.

Degree of a vertex :—

The degree of a vertex in a graph with the number of edges incident with it.

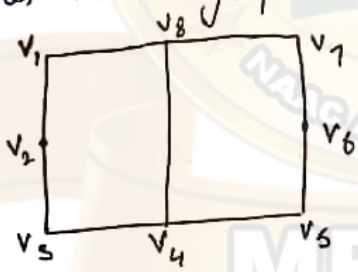
Note :- If G = (V, E) has an Eulerian path then it is connected. the edge {a, b} will contribute degree 1 to a. and degree 1 to b.

If the loop {a, a} will contribute to 1 to a and 2 to a.

Hamiltonian Path / circuits :—

A Hamiltonian Path or circuit in a graph G is a path or circuit that transverse each of the vertices in the graph G. Exactly once.

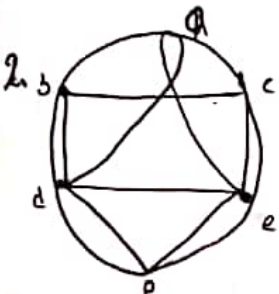
Ex :— 1.



The graph has a hamiltonian path or ~~circuit~~ circuit is

$v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9$.

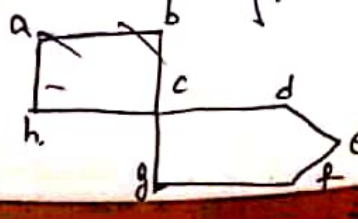
2.



This graph has an Eulerian circuit and also hamiltanion circuit, the circuit is a, b, d, e, f, c, a.

It is an example for both having hamiltanion and Eulerian circuit and also individually.

3.



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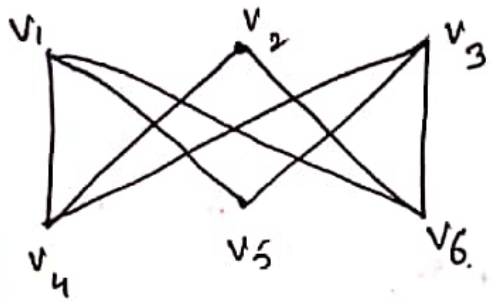
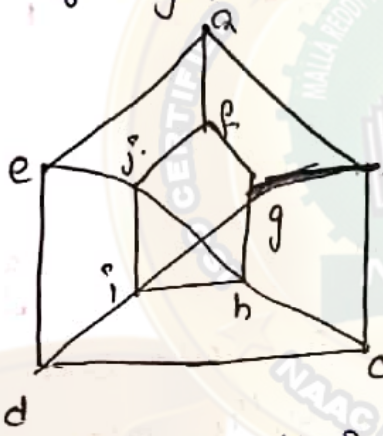
This is an Example for Eulerian circuit but not Hamiltanion circuit as it also exist the Eulerian path.

4. 

This graph has Hamiltanion Path and points coincides. So it is a Hamiltanion circuit and i.e., $V_4, V_1, V_5, V_2, V_6, V_3, V_4$.

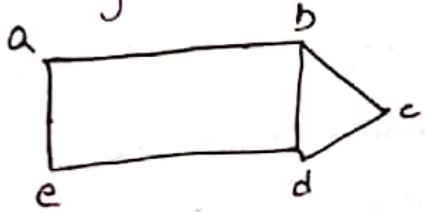
It is not possessing Eulerian circuit but possiging the Hamiltanion because all of its vertices having odd degree of edges.

5. 

It is not Eulerian circuit and not Eulerian path. this graph is an Example for neither possing Eulerian circuit not possing Hamiltanion circuit.

Linear Graph :— Linear graph which is not a Multi graph.

Note :— Let G be a linear graph with n-vertices if the sum of the degree for each pair of vertices is n-1 for larger then their exists a Hamiltanion Path.



It is sufficient condition but not necessary.

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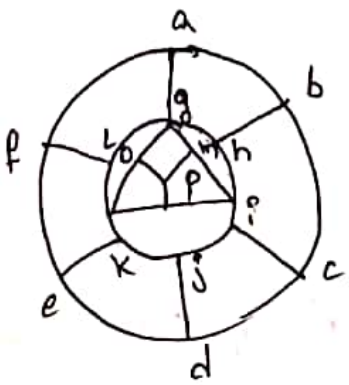
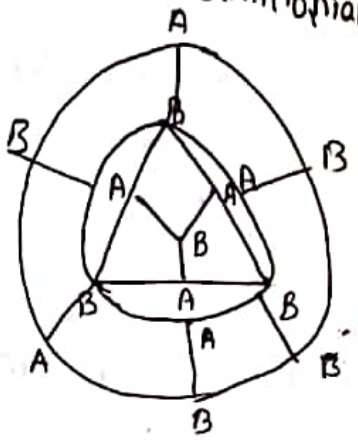
Problems:—

1. Show that the following graph has no Hamiltonian Path.



&



Sol:- Label the vertex a by A and Label all the vertices that are adjacent to it by B. Label all the vertices that are adjacent to a. Label B by A, until all the vertices are labelled. If there is an Hamiltonian path then it should pass through A and B alternatively since there are 9A's and 7B's.

Thus Existance of an Hamiltonian Path is not possible.—

Note:- There is an Hamiltonian path in a complete directed graph. (a graph must always possess a Hamiltonian Path).

Special graph:—

cycle graph:— A cycle graph of order n is a connected graph whose Edges form a cycle of length n. cycle graph of order n. denoted by C_n.

Ex:—

Path graph:— A Path graph of Order n is obtained by moving an Edge from a cycle graph of Order n' is denoted by P_n.

Ex:—

P_5

Wheel graph:— A wheel graph of Order n is a graph obtained by joining in new vertex to Each vertex of a cycle. graph of Order n' and is denoted by "W_n".

Ex:—

W_6

Null graph:— A Null graph of Order n' is a graph with n vertices and no. of Edges and is denoted by N_n.

Ex:—

N_4

Biparatite Graph:— A Biparatite graph is a non-directed graph in which the Set of vertices Can be. Partition Into two Sets M and N in Such a way that Each Edge joints a vertex in M to a vertex. N.

Ex:—

Biparatite graph.

not Biparatite graph

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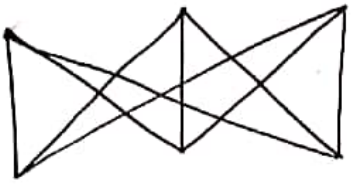
Complete Bipartite Graph:— A complete bipartite graph is a Bipartite graph in which every vertex of M is a adjacent to a vertex of N.

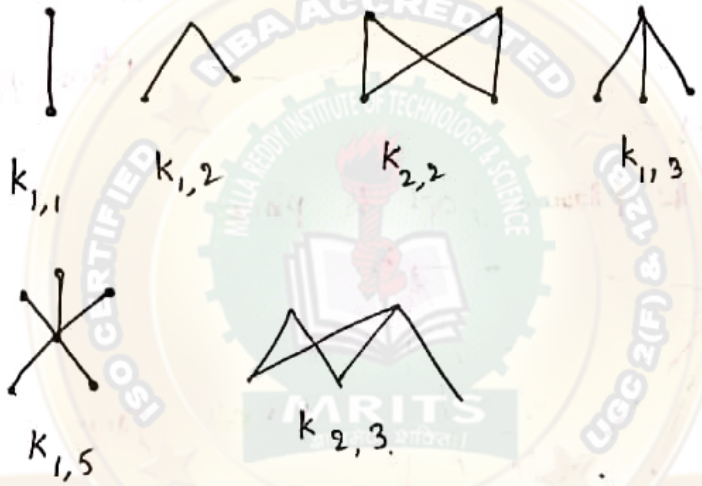
Ex:-



The graph $K_{3,3}$ is a C.B.G.

Note:- If $|M| = m$, $|N| = n$, then the complete Bipartite graph is denoted by $K_{m,n}$.

Ex:-



Star graph:— Any complete Bipartite graph $K_{1,n}$ is called Star graph.

Ex:-

Planar graph:— A graph is said to be Planar graph if it can be drawn on a plane without only cross otherwise G is said to non-planar graph.

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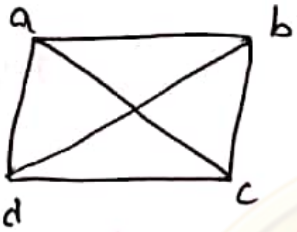
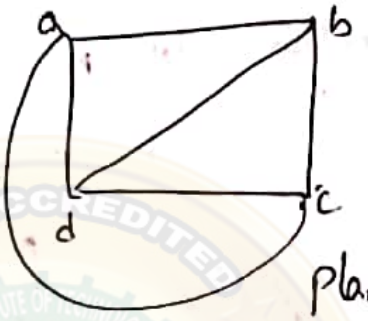
Note:— If a graph G is drawn with crossing Edges this doesnot mean that G is not planar. their may be another way to draw the graph without cross over.

→ A planar graph is a plane graph if it is already drawn in the plane so that no two Edges cross out.

Ex:—

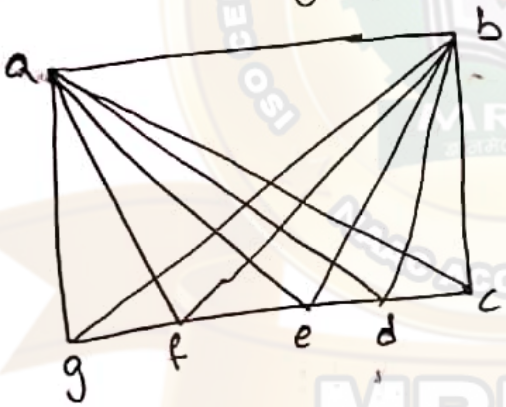




plane graph.

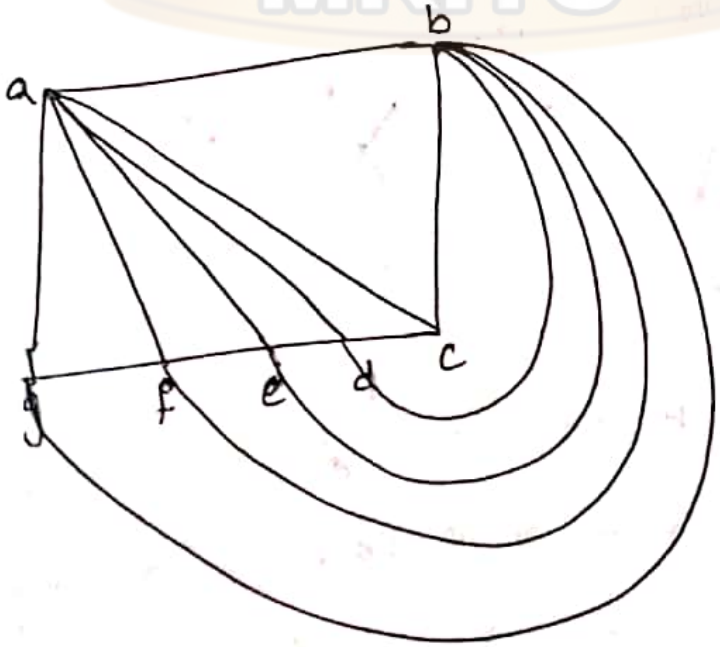
Problems:—

1) Q.? the following graph is planar.



planar graph.

Sol:—



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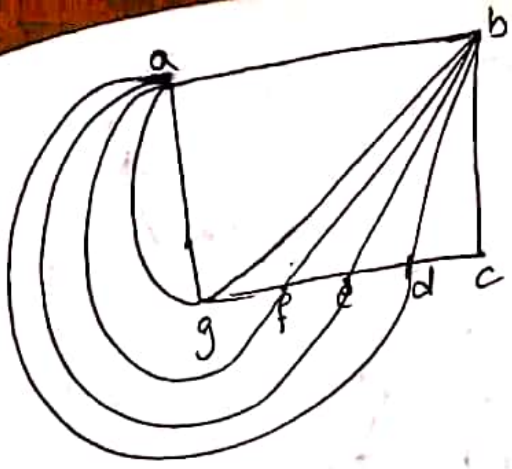
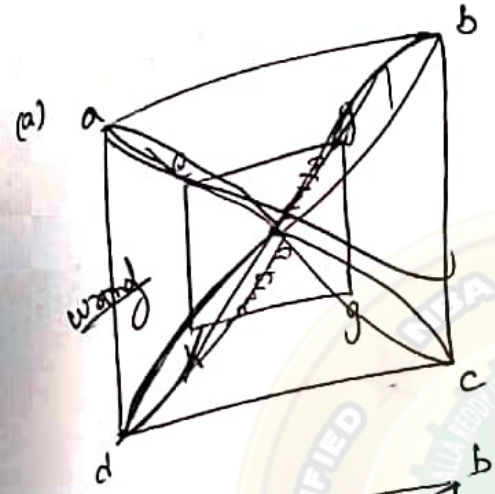
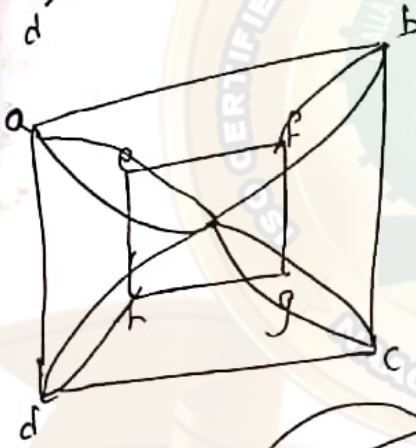


(a)

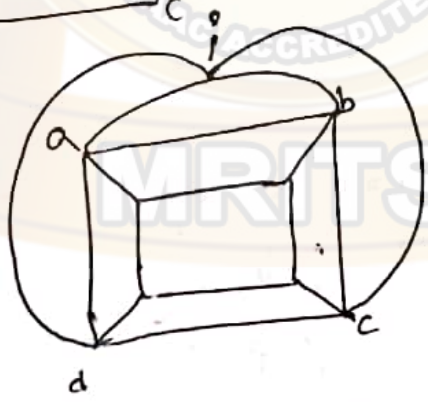


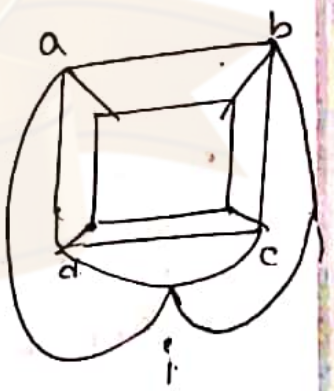
(b)

(a)

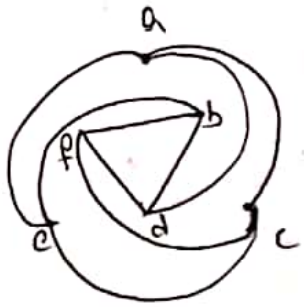


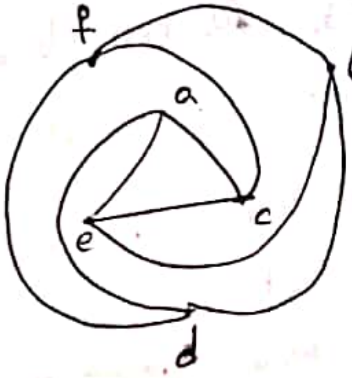
Sol:- (a) :-





(b)





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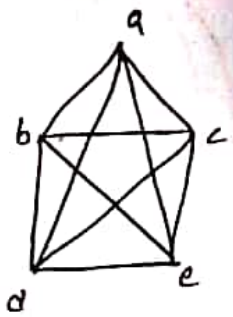
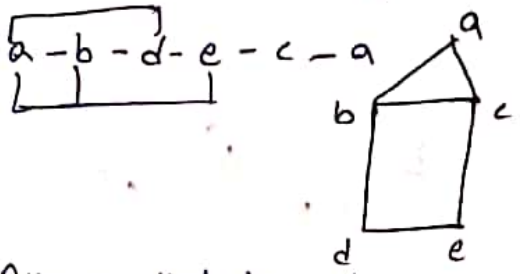
1) S.T. k_5 is non-planar.

Sol:- Consider k_5 graph

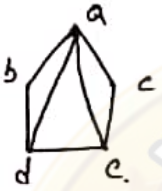


first we draw a cycle.



Assume that the edge {a,d} {a,e} inside the circle.

i.e.,



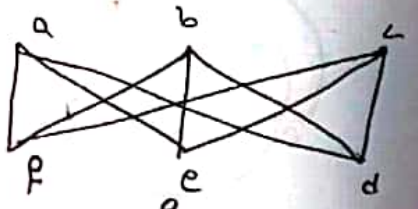
The edge {b,c} {b,e} must be drawn out the circle otherwise there would be cross of it. with the Edge {a,d}, {a,e}

But there is no place the edge {d,c} to draw either inside or outside the cycle.

∴ Hence the given k_5 graph is non-planar.

2. P.T. $k_{3,3}$ is non planar graph.

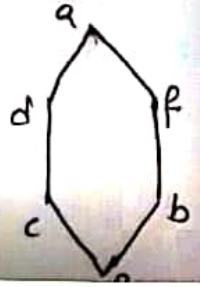
Sol:- Consider $k_{3,3}$ graph.



first we draw cycle of

a - d - c - e - b - f - a.

we can assume that the edge {a,e} inside the cycle.



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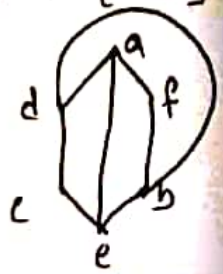
and the Edge {d,b} must be drawn outer to the cycle. Otherwise would be cross off b/w {a,e} and {b,d}

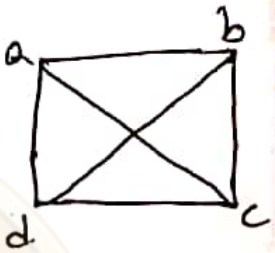
i.e., but there is no place to draw the edge {c,f} either inside or outside cycle.



∴ Hence $K_{3,3}$ is non-planar.

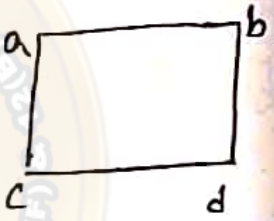
3. S.T. K_4 - graph is planar.

Proof: Consider K_4 - graph is



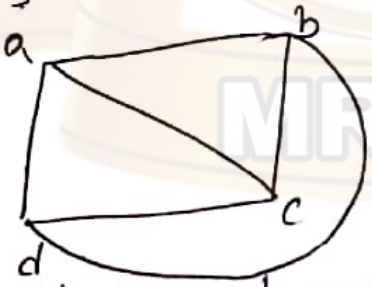
first we draw a cycle of

a - b - c - d - a.



Assume that the Edge {a,c} inside. the cycle and the Edge {b,d} must be drawn outside the cycle otherwise would be cross offs. b/w {a,c} and {b,d}

i.e.,



Hence K_4 is a planar.

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